Foreign Vulnerabilities, Domestic Risks: The Global Drivers of GDP-at-Risk*

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Abstract

We study how foreign financial developments influence the conditional distribution of domestic GDP growth. We propose a method to account for foreign vulnerabilities using bilateral-exposure weights when assessing downside macroeconomic risks within quantile regressions. For an advanced-economy panel, we show that tighter foreign financial conditions and faster foreign credit-to-GDP growth are associated with a more severe left-tail of domestic GDP growth, even controlling for domestic indicators. Incorporating foreign variables improves estimates of domestic GDP-at-Risk, both in and out of sample. Decomposing GDP-at-Risk into domestic and foreign origins, we show that foreign shocks are a key driver of domestic macroeconomic tail risks.

JEL Codes: E44, E58, F30, F41, F44, G01.

Key Words: Financial stability; GDP-at-Risk; International spillovers; Local projections; Quantile regression; Tail risk.

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1 Introduction

It is well established that *domestic* financial developments can generate downside risks to *domestic* economic growth (Adrian, Boyarchenko, and Giannone, 2019; Aikman, Bridges, Hacioglu Hoke, O'Neill, and Raja, 2019) and, in turn, can influence the probability of crises (Schularick and Taylor, 2012). But not all crises have domestic origins. In a highly interconnected and increasingly synchronized global economy, international vulnerabilities can spill over to the domestic risk environment.[1] But how, and to what extent, does this occur?

In this paper, we document the crucial role of *foreign* vulnerabilities in determining downside risks to *domestic* economic growth. Tighter foreign financial conditions and faster foreign credit-to-GDP growth can generate significant macroeconomic tail risks, even when controlling for domestic indicators. In particular, we show that they weigh heavily on 'GDP-at-Risk'—the 5th percentile of the GDP-growth distribution. A summary measure of downside macroeconomic risks, GDP-at-Risk is a now widely used concept in financial-stability monitoring and cost-benefit analysis informing macroprudential policy (Carney, 2020).

Foreign financial developments can influence domestic GDP-at-Risk, and the conditional distribution of domestic GDP growth more generally, through a number of channels. First, consistent with evidence of a global financial cycle (Rey, 2013; Miranda-Agrippino and Rey, 2020) characterized by strong cross-country comovement in asset prices, a substantial portion of variation in domestic financial conditions can arise from common global sources. Tighter global financial conditions can impact domestic funding costs and risky asset prices, and, in turn, the conditional distribution of future GDP growth outturns. Second, with financial institutions increasingly holding foreign claims, excessive credit growth and risk taking abroad can generate losses for domestic financial institutions and cause spillovers to the wider economy. Third, a build-up in foreign vulnerabilities that triggers a downturn abroad can spill over to the domestic economy through broader macroeconomic channels—for instance by lowering demand for domestic exports. While the influence of foreign factors on the *mean* of domestic GDP growth is widely studied in the international business-cycle literature (see Corsetti, 2008, and the references within), their influence on the *tails* of the domestic GDP growth distribution is the subject of this paper.

In our key methodological contribution, we propose a general and parsimonious approach to account for the influence of foreign vulnerabilities on the conditional distribution of domestic GDP growth. We do so within a quantile regression setup (Koenker and Bassett, 1978) that allows us to estimate the relationship between a range of indicators and the GDP-growth distribution over time and across countries. We account for foreign vulnerabilities by defining a weighted average of indicators in the rest of the world using bilateral-exposure weights. This

[1] See, for example, Cesa-Bianchi, Dickinson, Kösem, Lloyd, and Manuel (2021) for a summary of the channels through which these cross-border spillovers can occur.

approach has the advantage of capturing country-specific exposures to foreign vulnerabilities, while also limiting the number of additional regressors—a particular computational challenge for quantile regression.

We then apply this methodology to a cross-country panel dataset of advanced economies. Doing so provides novel empirical evidence demonstrating the link between foreign vulnerabilities and domestic GDP-at-Risk, as well as the conditional distribution of GDP growth. We emphasize three main findings.

First, we show that foreign vulnerabilities significantly and robustly influence the conditional distribution of future domestic GDP growth, even when controlling for domestic indicators. Tighter foreign financial conditions are associated with significant reductions in the left tail of domestic GDP in the near term—i.e., less than 1 year. Faster foreign credit-to-GDP growth weighs on the 5th percentile of domestic GDP growth out to longer horizons—i.e., up to 5 years. Moreover, the influence of these foreign variables on the distribution of domestic GDP growth is significantly larger at the 5th percentile than at the median, indicating that global financial developments can have non-linear impacts on domestic GDP.

Second, we demonstrate that foreign indicators provide information relevant for estimating domestic GDP-at-Risk, over and above domestic ones, both in and out of sample. The inclusion of foreign vulnerabilities significantly improves estimates of domestic GDP-at-Risk. The in-sample goodness-of-fit for estimates of the 5th percentile of domestic GDP are materially higher when foreign-weighted variables are included in the quantile regression specification, even when excluding the period containing the 2007-2008 Global Financial Crisis (GFC). We also find evidence of improved out-of-sample performance from including foreign variables, albeit limited to near-term horizons. By capturing vulnerabilities relevant for the tails of the GDP-growth distribution, we also show that foreign indicators can help to improve the narrative around higher-order moments estimated within a quantile regression framework. This highlights the importance of monitoring global variables when assessing macroeconomic tail risks.

Finally, we move towards a structural decomposition of historical estimates of GDP-at-Risk by orthogonalizing domestic variables with respect to foreign ones—an approach we show to be equivalent to a factor model. This allows us to identify and estimate the substantial contribution of foreign shocks to domestic macroeconomic tail risks. On average, we show that foreign shocks explain up to around 90% of variation in the estimated 5th percentile of advanced-economy GDP growth over the 1-year horizon, more than the comparable figure for the median.

Our results have important implications for financial stability policy. By highlighting the additional explanatory power of foreign variables to domestic GDP-at-Risk, we show the importance of accounting for foreign indicators when monitoring risks to domestic financial stability. In addition, by demonstrating the substantial contribution of foreign shocks to domestic tail risks, our results suggest that international macroprudential policy frameworks that foster cooperation between national authorities when forming regulatory responses to global shocks can be beneficial. More broadly, our general methodology can be applied more widely, for instance to inform analyses of GDP-at-Risk within emerging-market economies, where assessments of tail risks have been more limited in spite of their substantial exposures to foreign events. Such analyses could shed further light on the role of macroprudential policy in guarding against tail risks in the face of foreign shocks (e.g., Coman and Lloyd, 2022).

Related Literature Our paper is related to three main strands of literature. First, and most directly, our work builds on studies applying quantile regression techniques to assess the drivers of macroeconomic tail risks (see, e.g., Adrian, Boyarchenko, and Giannone, 2019; Adrian, Grinberg, Liang, Malik, and Yu, 2022; Aikman, Bridges, Hacioglu Hoke, O'Neill, and Raja, 2019).[2] Using data on advanced economies, these papers identify a strong relationship between *domestic* vulnerabilities, such as financial conditions and credit growth, and the tails of the conditional GDP growth distribution. But they do not explicitly account for the influence of *foreign* vulnerabilities. These will only be implicitly captured insofar as foreign vulnerabilities are reflected within domestic indicators. We contribute to this body of work by exploring the independent influence of foreign vulnerabilities, and propose a novel methodological framework for doing so.[3]

Second, our study relates to a literature on financial crisis warning indicators. Building on Schularick and Taylor (2012), who find credit-to-GDP to be a robust predictor of financial crises, others have shown that foreign variables can have significant predictive power. For instance, Cesa-Bianchi, Eguren-Martin, and Thwaites (2019a) and Bluwstein, Buckmann, Joseph, Kang, Kapadia, and Simsek (2020) find that *global* financial developments influence the probability of *domestic* crises, over and above domestic indicators. Our analysis extends this literature by documenting the influence of foreign factors on the whole conditional distribution of GDP growth—not just crisis events. Specifically, we also show that the information in foreign variables aids the narrative around higher-order moments of the GDP-growth distribution estimated within a quantile regression setup. For example, in the the run-up to the GFC, a model that includes foreign variables estimates a clear rise in variance and worsening in downside skew of GDP growth, and more so than models with only domestic covariates. These findings

[2]In part motivated by these papers, there have been a number of other studies of GDP tail risks using quantile regressions. For example: Giglio, Kelly, and Pruitt (2016) for the United States (US) and Europe, Aikman, Bridges, Burgess, Galletly, Levina, O'Neill, and Varadi (2018) for the United Kingdom (UK), Loria, Matthes, and Zhang (2019) for the US, Chavleishvili and Manganelli (2019) and Lhussier (2022) for the euro area, Duprey and Ueberfeldt (2020) for Canada, and Busetti, Caivano, Delle Monache, and Pacella (2021) for Italy. Others have proposed the use of quantile regression tools for high-frequency GDP-at-Risk monitoring (e.g., Ferrara, Mogliani, and Sahuc, 2022).

[3]Busetti et al. (2021) find a significant association between Italian GDP-at-Risk and US financial conditions, as well as a global purchasing managers' index. While this demonstrates some role for global factors in the determination of macroeconomic tail risks, the method we propose is more general and—as we go onto explain—has a number advantages over simply adding US variables, or global aggregates, to the explanatory-variable set.

relate to recent work by Plagborg-Møller, Reichlin, Ricco, and Hasenzagl (2020) discussing the interpretability of estimated higher-order moments within a quantile regression framework.

Finally, our paper has links with the broad literature on disaster risks and economic growth (see, e.g., Barro, 2009; Barro and Ursúa, 2012; Gabaix, 2012; Gourio, 2012; Wachter, 2013). In particular, our evidence emphasizing the importance of foreign vulnerabilities for domestic downside risks contributes to recent work highlighting the cross-border transmission of macroeconomic disasters (Gourio, Siemer, and Verdelhan, 2013; Farhi and Gabaix, 2016).

The remainder of this paper is structured as follows. Section 2 presents our general methodology. Section 3 describes the results from a specific application, emphasising the additional information foreign variables provide over and above domestic ones in and out of sample. Section 4 moves towards a structural assessment, decomposing GDP-at-Risk estimates into domestic and foreign shocks. Section 5 concludes.

2 Methodology to Account for Global Drivers

In this section, we outline our general methodology to account for global drivers of GDP-at-Risk and the conditional distribution of GDP growth. As in previous work, we employ a quantile regression framework (Koenker and Bassett, 1978) to study how changes in a set of conditioning variables are associated with the distribution of future GDP growth. We present our approach within a panel setting, where time is denoted by $t = 1, ..., T$ and the countries for whom we estimate the conditional distribution of GDP are labelled with $i = 1, ..., N$.[4]

We specify the following model for the conditional quantile function Q of h-period-ahead country-i GDP growth $\Delta^h y_{i,t+h}$:

$$Q_{\Delta^h y_{i,t+h}}(\tau|\mathbf{x}_{i,t}, \mathbf{x}^*_{i,t}) = \alpha^h_i(\tau) + \mathbf{x}'_{i,t}\boldsymbol{\beta}^h(\tau) + {\mathbf{x}^*_{i,t}}'\boldsymbol{\vartheta}^h(\tau) \tag{1}$$

where Q computes quantiles τ of the distribution of $\Delta^h y_{i,t+h}$ given covariates: $\mathbf{x}_{i,t}$ and $\mathbf{x}^*_{i,t}$, both $K \times 1$ vectors, with associated $K \times 1$ parameter vectors $\boldsymbol{\beta}^h(\tau)$ and $\boldsymbol{\vartheta}^h(\tau)$, respectively. In stacked notation, where $\mathbf{y}_{t+h} = [Q_{\Delta^h y_{1,t+h}}, ..., Q_{\Delta^h y_{N,t+h}}]'$ is an $N \times 1$ vector, $\boldsymbol{\alpha}^h(\tau) = [\alpha^h_1(\tau), ..., \alpha^h_N(\tau)]'$ is an $N \times 1$ vector, and $\mathbf{X}_t = [\mathbf{x}'_{1,t}, ..., \mathbf{x}'_{N,t}]'$ and $\mathbf{X}^*_t = [{\mathbf{x}^*_{1,t}}', ..., {\mathbf{x}^*_{N,t}}']'$ are $N \times K$ matrices, this expression can be written as:

$$\mathbf{y}_{t+h} = \boldsymbol{\alpha}^h(\tau) + \mathbf{X}_t\boldsymbol{\beta}^h(\tau) + \mathbf{X}^*_t\boldsymbol{\vartheta}^h(\tau) \tag{2}$$

$\alpha^h_i(\tau)$ represents a country- and quantile-specific fixed effect to control for time-invariant unobserved heterogeneity. Estimation of the panel quantile regressions with quantile-specific country fixed effects is feasible when the panel structure has T much larger than N (Galvao

[4]Our general approach to accounting for global factors can also be applied to country-specific regressions—as we explain in robustness analysis in Section 3.

and Montes-Rojas, 2015), as is the case in our application.[5] In this $T \gg N$ case, Kato, Galvao, and Montes-Rojas (2012) demonstrate that this panel fixed-effects estimator is consistent and asymptotically normal, a finding verified using a different approach by Galvao, Gu, and Volgushev (2020).[6]

The domestic covariates in equation (1) are denoted by $\mathbf{x}_{i,t}$. They include domestic indicators that may influence the conditional distribution of domestic GDP, such as credit growth or proxies for financial conditions. The key novelty in equation (1) is the inclusion of foreign covariates $\mathbf{x}^*_{i,t}$. These foreign indicators can reflect both foreign country-specific factors and common global events. However, as we explain in the next sub-section, the construction of these foreign variables is not trivial.

2.1 Constructing Foreign Covariates

To appreciate these challenges, consider a country $i \in [1, N]$ for whom we estimate the conditional distribution of GDP growth using equation (1). The τ-th quantile of GDP growth in country i can depend on domestic covariates $\mathbf{x}_{i,t}$, but also a set of indicators $\mathbf{x}_{j,t}$ in a range of other countries $j = 1, ..., N^*$.

In order to account for the influence of a single foreign indicator (e.g., credit-to-GDP) on the conditional distribution of domestic (country-i) GDP, one approach could be to individually add this indicator for each foreign country $j = 1, ..., N^*$, where $j \neq i$, to the foreign-covariate set $\mathbf{x}^*_{i,t}$. However, this would lead to a proliferation of regressors, adding an extra $N^* - 1$ explanatory variables. This could pose computational challenges for the quantile regression—especially if scaled up to more than one foreign indicator—and, in the limit, would exhaust available degrees of freedom.[7]

To circumvent this *curse of dimensionality*, for each indicator (e.g., credit-to-GDP) we define a single foreign covariate $x^*_{i,t} \subset \mathbf{x}^*_{i,t}$ as the weighted sum of the indicator $x_{j,t}$ in all other countries $j = 1, ..., N^*$. Defining $\omega_{i,j,t}$ as a time-varying weight capturing the *bilateral exposure* of country i to country j at time t, we construct the foreign-weighted sum for each indicator using:

$$x^*_{i,t} = \sum_{j=1}^{N^*} \omega_{i,j,t} x_{j,t} \tag{3}$$

[5]See Lamarche (2021) for a recent survey of panel quantile regression estimators.

[6]Our approach to account for foreign vulnerabilities does not depend on specific assumptions about the constant term. Our main results are robust to using an alternative country fixed-effects structure, in which the fixed effect is the same across quantiles for a given country, i.e., α_i^h for all τ, alongside a quantile-specific intercept (Canay, 2011).

[7]One option for dealing with high-dimensional data of this kind may be to employ the penalized quantile regression of Wu and Liu (2009). Relative to our approach, the penalized quantile regression does not impose any structure on cross-country linkages. While this has the advantage of *letting the data speak*, in practice the cross-country linkages it estimates can be implausible and out-of-line with standard narratives around the propagation of spillovers. Future research could seek to balance the advantages of this purely data-driven approach with the more structured approach implied by our imposed weighting scheme.

where $\omega_{i,j,t} \geq 0$ for all i, j, t, as well as $\sum_{j=1}^{N^*} \omega_{i,j,t} = 1$ and $\omega_{i,i,t} = 0$ for all i, t. Alternatively, in the stacked notation of equation (2), this definition can be stated as:

$$\mathbf{X}_t^* = \mathbf{W}_t \mathbf{X}_t \tag{4}$$

where W_t is an $N \times N^*$ matrix collecting the $\omega_{i,j,t}$ with i denoting rows and j columns.

With this definition, each additional foreign indicator (e.g., credit-to-GDP) adds a single regressor (e.g., foreign-weighted credit-to-GDP) to equation (1), offering a parsimonious solution to the curse of dimensionality. Furthermore, by constructing the foreign covariates in this way, we can extend the number of foreign countries N^* that we account for, without increasing dimensionality. There is also no restriction that the number of foreign countries N^* needs to be the same as the number of domestic ones N.[8]

Moreover, by using weights $\omega_{i,j,t}$ that capture *country-specific* bilateral exposures to the rest of the world, we account for heterogeneity in countries' cross-border links. For instance, we can ensure that countries with stronger ties to country i through trade or financial linkages (i.e., larger $\omega_{i,j,t}$) comprise a larger share of the foreign-weighted covariate and therefore can have a stronger association with the conditional distribution of country-i GDP growth. This desirable economic intuition would be lost were we to specify each $x_{i,t}^*$ as a simple global aggregate (e.g., global credit-to-GDP), i.e., the sum (or unweighted average) of country-j indicators.

In addition, our proposal nests an approach in which only US variables (e.g., US VIX) are used to capture global events (i.e., $\omega_{i,US,t} = 1$ and $\omega_{i,j,t} = 0 \ \forall \ t, \ j \neq US$). While such a US-specific setup can capture elements of the global financial cycle emanating from the US, our proposal allows for a broader set of cross-border transmission channels and shocks, including the build-up of regional risks (e.g., within the euro area). Moreover, relative to a US-only foreign variable, which is homogeneous for all countries within the panel, our foreign-weighted variable is heterogeneous across countries. So, in more general settings, equation (3) can be used alongside fixed effects that are homogeneous with respect to i (e.g., time fixed effects).

The coefficients $\boldsymbol{\beta}^h(\tau)$ denote the average association between domestic covariates and quantiles τ of the GDP-growth distribution. The coefficients $\boldsymbol{\vartheta}^h(\tau)$ represent the average association between *foreign* indicators and the conditional distribution of *domestic* GDP growth.

Within our setup, the coefficients denote the average association between each covariate and quantiles τ of the GDP-growth distribution, holding all other covariates (both domestic and foreign) fixed. Specifically, $\beta_k^h(\tau)$ represents the average association between the k-th domestic covariate in $\mathbf{x}_{i,t}$ and the τ-th percentile of h-period-ahead domestic GDP growth, holding foreign covariates as well as other domestic covariates fixed. Similarly, $\vartheta_k^h(\tau)$ represents the average association between the k-th foreign covariate in $\mathbf{x}_{i,t}^*$ and the dependent variables,

[8] For instance, we may estimate GDP-at-Risk for a set of N similar advanced economies, but want to account for spillover channels from a broader set of countries $N^* > N$, which may include major emerging markets in addition to the N advanced economies.

holding domestic covariates and other foreign covariates constant.[9] To aid with intuition, we can also interpret the coefficients in the following manner for globally-synchronized events (i.e., when a covariate k changes in all countries simultaneously) drawing on the spatial econometrics literature (see, e.g., Debarsy, Ertur, and LeSage, 2012). Here, $\beta_k^h(\tau)$ represents the *direct effect* of global events on domestic growth-at-risk, while $\vartheta_k^h(\tau)\mathbf{W}_t$ represents *indirect effects*.[10]

Overall, our approach is parsimonious, while also maintaining a meaningful economic narrative around cross-country links. As in the global vector autoregression (GVAR) literature, where similar weighting schemes are applied to account for the influence of foreign factors at the mean (Pesaran, Schuermann, and Weiner, 2004; Eickmeier and Ng, 2015), there are a number of candidate weighting schemes that can be used too. For example, weights can be constructed based on bilateral trade or financial linkages (or combinations thereof) depending on practitioners' focus.

3 Documenting the Global Drivers

In this section, we estimate the global drivers of the conditional distribution of GDP growth, emphasising the additional information provided by foreign indicators, in and out of sample.

3.1 Empirical Specification

We illustrate our general methodology with a specific empirical model. This model is similar to the specification in Adrian et al. (2022) and is deliberately pared back, in order to highlight the influence of the key global drivers of the conditional distribution of GDP growth. However, as we emphasize in subsequent sub-sections, our key findings are robust to a range of alternative model specifications, reflecting the generality of our approach.

As is common within the growth-at-risk literature (e.g., Aikman et al., 2019; Franta and Gambacorta, 2020; Galán, 2020; Adrian et al., 2022), we estimate the conditional distribution of GDP growth for a panel economies in our baseline specification, specifically for 10 advanced economies in our setting: Australia, Canada, France, Germany, Italy, Spain, Sweden, Switzerland, UK and US. The dataset spans the period 1981Q1 to 2016Q3.[11] Our dependent variable is formally defined as annual average real GDP growth over h quarters, i.e., $\Delta^h y_{i,t+h} \equiv (y_{i,t+h} - y_{i,t})/(h/4)$. As well as providing additional data with which to robustly estimate coefficients in sample, estimating our model for a panel of countries can also bring efficiency gains out of sample (as we come on to demonstrate). Our setup can be also extended to

[9] To see this, let $x_{i,t}^{(k)}$ denote the k-th element of $\mathbf{x}_{i,t}$, $\beta_k^h(\tau)$ the k-th element of $\boldsymbol{\beta}^h(\tau)$ and $\vartheta_k^h(\tau)$ the k-th element of $\boldsymbol{\vartheta}^h(\tau)$. We can show that for all k: $\frac{\partial Q_{\Delta^h y_{i,t+h}}}{\partial x_{i,t}^{(k)}} = \beta_k^h(\tau)$ and $\sum_{j\neq i} \frac{\partial Q_{\Delta^h y_{i,t+h}}}{\partial x_{j,t}^{(k)}} = \vartheta_k^h(\tau)\sum_{j\neq i}\omega_{i,j,t} = \vartheta_k^h(\tau)$.

[10] To see this, substitute equation (4) into (2), and let $\mathbf{x}_t^{(k)}$ denote the k-th column of $\mathbf{X}_t$ (i.e., the k-th element of $\mathbf{x}_{i,t}$ for all countries $i = 1, ..., N$), to yield: $\frac{\partial \mathbf{y}_{t+h}}{\partial \mathbf{x}_t^{(k)}} = \beta_k^h(\tau)\mathbf{I}_N + \vartheta_k^h(\tau)\mathbf{W}_t$.

[11] See Appendix A for a full description of data sources.

account for potential cross-country heterogeneity via the use of interaction terms as we discuss in Section 3.2.2.

Domestic Covariates We include three domestic indicators in the variable set $\mathbf{x}_{i,t}$: (i) a financial conditions index (FCI); (ii) the 3-year percentage point change in the aggregate private non-financial credit-to-GDP ratio; and (iii) the 1-quarter growth of real GDP.[12] This variable choice is motivated by existing studies focusing on domestic GDP-at-Risk (see, e.g., Aikman et al., 2019; Adrian et al., 2022). Like Adrian et al. (2022), we use a FCI constructed per the method of Koop and Korobilis (2014). This index is a summary measure that extracts common variation across a range of asset prices.[13] We favor the 3-year change in credit-to-GDP to capture *persistent* changes in credit, which are thought to pose risks to financial stability and are leading indicators of macroeconomic crises (Schularick and Taylor, 2012). Moreover, we choose to separate these two vulnerability indicators—rather than use a single aggregated indicator of price- and quantity-based vulnerabilities, as in Plagborg-Møller et al. (2020) for example—to capture the differing influence of risk factors across horizons. As Aikman et al. (2019) and Adrian et al. (2022) show, tighter financial market conditions tends to have a negative near-term influence on the left-tail of GDP growth, while growth in the quantity of credit relative to GDP is associated with a medium-term deterioration in GDP-at-Risk. Quarterly real GDP growth is included as a control for the prevailing state of the macroeconomy.

Foreign Covariates We include the foreign-weighted counterparts of each of the three indicators in the foreign variable set $\mathbf{x}^*_{i,t}$. This variable choice is, in part, motivated by evidence that global financial market indicators and credit quantities separately tend to predict domestic financial crises (Cesa-Bianchi et al., 2019a; Bluwstein et al., 2020). The foreign-weighted FCI reflects asset price developments abroad, while the foreign-weighted 3-year change in credit-to-GDP captures changes in credit quantities in the rest of the world. Our baseline specification also includes foreign-weighted quarterly real GDP growth to account for global business cycle dynamics.[14]

For our baseline results, we construct foreign-weighted variables using data on bilateral trade linkages. Using data from IMF Direction of Trade Statistics, we define the weights $\omega_{i,j,t}$ as the fraction of country i's exports to country j at time t. This scheme will place higher weight on foreign regions that country i exports more extensively to, reflecting the fact that a downturn in one country j may spill over to another i through reduced demand for country-i exports. Compared to the bilateral financial weights from BIS International Banking Statistics

[12] We discuss the robustness of our findings to different specifications of domestic risk factors in Appendix B.2.

[13] Our results are robust to the use of an alternative measure of financial-market conditions, specifically near-term equity volatility as in Aikman et al. (2019), as Appendix B.2 demonstrates.

[14] Clark, Huber, Koop, Marcellino, and Pfarrhofer (2021), amongst others, demonstrate the importance of global business cycle dynamics for growth-at-risk.

we use in robustness analyses in Appendix B.2, these trade weights have the advantage of running back to 1980, enabling us to use time-varying weights in the baseline specification. However, as we discuss there, our key results are robust to different combinations of country weights. Moreover, owing to constraints on data availability, we focus on the same set of *foreign* countries used in the *domestic* variable set, i.e., $N = N^* = 10$.[15]

Interpretation and Inference For presentational purposes, we standardise all regressors by the country-level mean and standard deviation. So, all coefficients can be interpreted as the association between a one standard deviation change in an indicator and the τ-th quantile of GDP growth. We estimate regression (1) for $h = 1, 2, ..., 20$ quarters. For inference, we follow the block bootstrap procedure of Kapetanios (2008), resampling the data over blocks of different time series dimensions to generate coefficient standard errors for respective quantiles. As in Aikman et al. (2019), we resample time series observations using 8 blocks, replicating the bootstrap 5000 times.

3.2 In-Sample Results

Using this baseline model, we first present in-sample results, including coefficient estimates, measures of model fit, as well as estimates of higher moments and the distribution of GDP growth.

3.2.1 Coefficient Estimates

Table 1 compares coefficient estimates from our baseline domestic-only (Panel A) and foreign-augmented (Panel B) models for the 5th percentile and the median, across horizons, while Figure 1 presents coefficient estimates at the 5th percentile of GDP across horizons h for financial conditions and the 3-year change in credit-to-GDP. The top panels of Figure 1 demonstrate the association between domestic vulnerability indicators and domestic GDP-at-Risk from our baseline foreign-augmented model (blue) and, for comparison, we also present coefficient estimates from a domestic-only specification (red), which excludes the foreign-weighted variables from the regressor set.[16] The lower panels plot the association between foreign-weighted vulnerability indicators and domestic GDP-at-Risk.

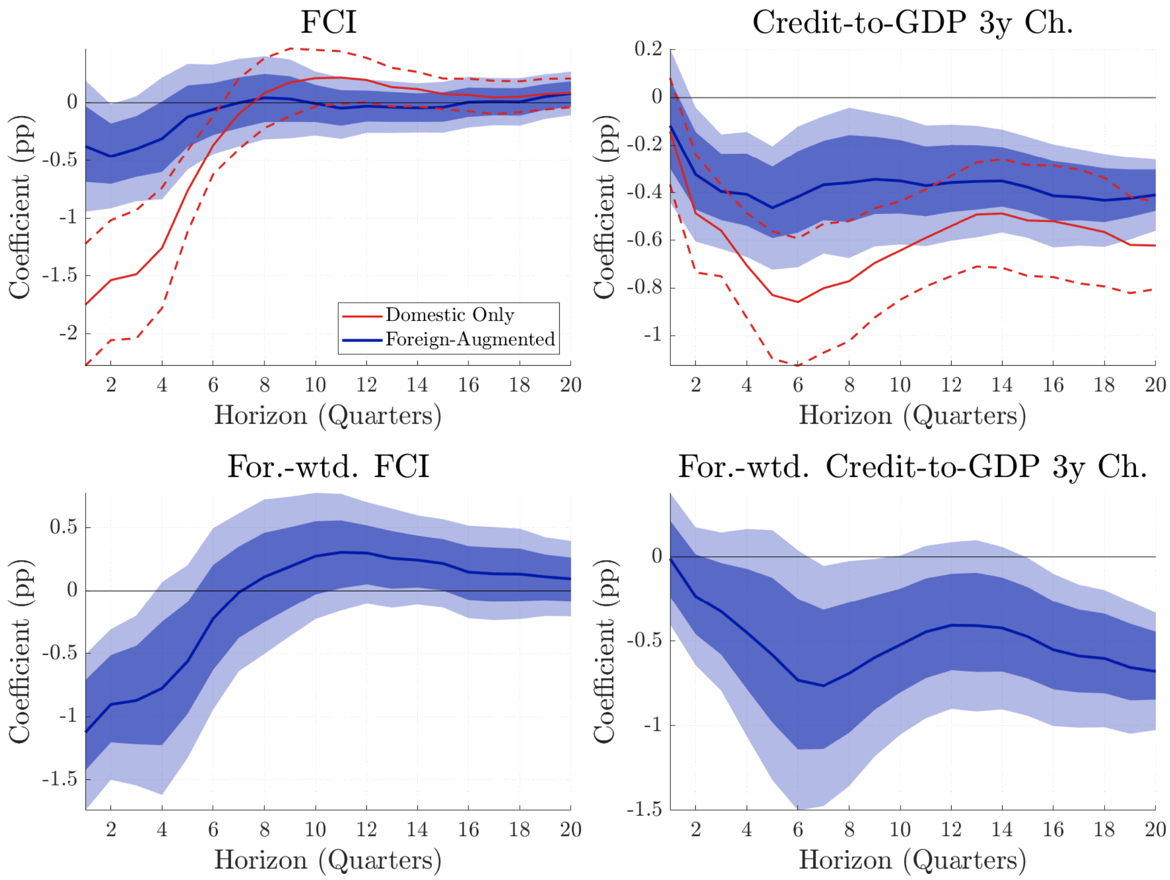
These results highlight the differing association between indicators and GDP-at-Risk across horizons and quantiles.[17] As in other studies (Aikman et al., 2019; Adrian et al., 2022), in the domestic-only specification (Panel A of Table 1 and red lines in upper panels of Figure 1),

[15] We discuss the robustness of our findings to a broader number of foreign countries ($N^* > N$) in Appendix B.2.

[16] Formally, the *domestic-only* model includes three covariates: (i) domestic FCI; (ii) domestic 3-year change in credit-to-GDP; and (iii) domestic quarterly real GDP growth.

[17] These coefficient estimates should not be strictly interpreted as causal given potential correlations between domestic and foreign-weighted covariates. We return to the issue of causality in Section 4, where we move towards a structural decomposition of the drivers of GDP-at-Risk.

Figure 1: Association between indicators and 5th percentile of GDP growth across horizons



Note: Estimated association between one standard deviation change in each indicator at time t with 5th percentile of annual average real GDP growth at each quarterly horizon. Solid red lines denote coefficient estimates from model that excludes foreign covariates and dashed red lines represent 90% confidence bands for these estimates from block bootstrap procedure. Solid blue lines denote coefficient estimates from model that includes foreign covariates. Light (dark) blue-shaded areas represent 90% (68%) confidence bands from block bootstrap procedure. Model also includes macroeconomic controls: domestic and foreign-weighted lagged quarterly real GDP growth.

tighter financial conditions weigh negatively on GDP-at-Risk in the near term—with the effect peaking in the first quarter, then waning over time. Higher domestic credit-to-GDP also has detrimental effects on the left-tail of GDP in the medium term—peaking at 6 quarters ahead and persisting out to year 5.

Financial Conditions Index The addition of foreign-weighted variables significantly alters the coefficient on domestic financial conditions (top-left panel) with its magnitude much reduced across horizons. At the 1-quarter horizon, a one standard deviation tightening in domestic financial conditions is associated with a 0.4pp deterioration in the 5th percentile of GDP growth (statistically significant at the 32% level), compared to a 1.7pp reduction in the domestic-only specification. In contrast, controlling for domestic FCIs, we find that tighter *for-*

Table 1: Coefficient Estimates from Baseline Model at 5th and 50th Percentile Across Horizons

	Horizons				
	h =1	h =4	h =8	h =12	h =20
	A: Domestic-Only Model				
Domestic Variables					
FCI	-1.749***	-1.258**	0.083	0.196^	0.085
	[-0.416***]	[-0.176^]	[-0.102]	[-0.085]	[0.032]
Credit-to-GDP	-0.142	-0.705***	-0.771***	-0.539**	-0.622***
	[-0.107]	[-0.192^]	[-0.367**]	[-0.483***]	[-0.542***]
GDP growth	0.716***	0.562***	0.219^	0.095^	0.068
	[0.788***]	[0.507***]	[0.237**]	[0.085^]	[0.016]
	B: Foreign-Augmented Model				
Domestic Variables					
FCI	-0.376^	-0.310	0.043	-0.029	0.080
	[-0.313**]	[-0.193^]	[-0.144^]	[-0.141^]	[-0.002]
Credit-to-GDP	-0.117	-0.407**	-0.358*	-0.357**	-0.409***
	[-0.160^]	[-0.190^]	[-0.322**]	[-0.382***]	[-0.415***]
GDP growth	0.255^	0.322*	0.211^	0.108^	-0.010
	[0.513***]	[0.317***]	[0.157*]	[0.043]	[-0.002]
Foreign Variables					
For. FCI	-1.125***	-0.775^	0.110	0.300^	0.095
	[-0.057]	[0.082]	[0.170^]	[0.217^]	[0.265^]
For. Credit-to-GDP	-0.011	-0.449^	-0.691*	-0.407^	-0.679***
	[0.102^]	[-0.013]	[-0.188^]	[-0.343^]	[-0.510***]
For. GDP growth	1.086***	0.418^	0.262	0.141^	0.106^
	[0.606***]	[0.378**]	[0.229^]	[0.163^]	[0.109^]
N (N^*)			10 (10)		
Weights (Sample)			Trade, Time-Varying (1981Q1-2016Q3)		

Note: Coefficient estimates from baseline model at 5th percentile [and median], with Panel A presenting estimates from model that excludes foreign covariates and Panel B showing estimates from model including foreign covariates. Significance, from block bootstrap, at 32%, 10%, 5% and 1% levels denoted by ^, *, ** and *** ,respectively.

eign financial conditions are associated with a large and significant near-term reduction in the left-tail of annual average domestic GDP growth (bottom-left panel). The 1-quarter coefficient indicates that a one standard deviation tightening in foreign-weighted financial conditions is, all else equal, linked with a 1.1pp fall in the 5th percentile of GDP growth (significant at the 1% level) that persists over the first year.

Moreover, there is some evidence that the association between foreign-weighted financial market conditions and the 5th percentile of GDP growth changes sign over horizons. This indicates that global financial market conditions can pose an inter-temporal trade-off for GDP-at-Risk: with tight financial conditions weighing negatively on the tails of growth near-term, but supporting growth-at-risk in the medium term by limiting potentially harmful risk taking. Adrian et al. (2022) note this trade-off in the context of their study. However, unlike them, our results suggest that the trade-off emanates from global financial conditions, not domestic ones.

Table 1 also highlights how the associations between the FCI and GDP growth vary across quantiles, particularly for the foreign-weighted indicator. In particular, the large and significant negative association between foreign FCIs and the 5th percentile is specific to the left tail. At the median, the near-term association is small and not statistically different from zero. In contrast, the differences across quantiles for domestic FCIs are less marked. Here, the median estimates are closer to coefficient estimates at the 5th percentile, indicating that—all else equal—changes in domestic financial conditions act broadly as a location shifter of the domestic GDP-growth distribution.

These estimates have parallels with findings in previous work, suggesting that financial-market uncertainty is more important for the business cycle when the shocks are global in nature (see, e.g., Eguren-Martin and Sokol, 2019; Cesa-Bianchi, Pesaran, and Rebucci, 2019b). This may reflect the fact that a shock that affects all countries at once can have particularly significant effects via global amplification mechanisms and non-linearities, which may not arise for a shock affecting the domestic economy only.

Credit-to-GDP The addition of foreign covariates has a less marked effect on estimates of the association between domestic credit growth and domestic tail risk (top-right panel). Even with foreign variables in the model, the domestic credit growth coefficient at the 5th percentile is negative at all horizons and, while its magnitude falls in the specification with foreign-weighted indicators, it remains significantly negative across horizons. At its peak, a one standard deviation increase in domestic credit-to-GDP is associated with a 0.4pp reduction in GDP-at-Risk in the foreign-augmented specification, compared to a reduction of around 0.8pp in the domestic-only specification.

Notwithstanding this, we also find that the association between foreign-weighted credit-to-GDP growth and domestic GDP-at-Risk is significantly negative in the near-to-medium term (bottom-right panel). The coefficient is significantly negative, at the 32% level at least from quarter-3 onward.[18] At its peak, a one standard deviation increase in foreign-weighted credit-to-GDP is associated with a 0.7pp reduction in the 5th percentile of GDP growth. So foreign credit-to-GDP growth appears to have similar quantitative effects on the domestic macroeconomic risk outlook as domestic credit-to-GDP.

The coefficient estimates in Table 1 also suggest significant non-linearities in the association across quantiles, again particularly for foreign indicators. The association between foreign credit and the median of the conditional GDP growth distribution is also negative at medium-term horizons (e.g., $h = 8$), but this negative effect is around a third of the estimated coefficient at the 5th percentile. As for financial conditions, while there is strong evidence of a non-linear relationship between foreign credit growth and domestic GDP growth across quantiles, the differences across quantiles for the coefficient on domestic credit growth are less marked. For instance, at $h = 8$, the coefficient estimate for domestic credit-to-GDP at the median is very similar to the point estimate at the 5th percentile.

[18] These point estimates are statistically significant at the 10% level for $h = 7$ to $h = 8$ and $h = 15$ onwards.

These findings have some parallels with the early-warning literature, where faster global credit growth has been found to be a significant predictor of crises (Lo Duca and Peltonen, 2013; Cesa-Bianchi et al., 2019a; Bluwstein et al., 2020). This literature finds that domestic and global credit growth have similar effects on the probability of a domestic banking crisis, similar to our findings in Figure 1. However, our results are more general, suggesting that this predictability arises specifically from the association between foreign credit growth and the left tail of the domestic GDP growth distribution.

There are a number of channels through which heightened credit growth abroad could affect downside tail risks to domestic GDP growth—even holding domestic credit growth fixed. Rapid credit growth abroad may increase the probability and severity of downturns in other countries, which in turn can influence the domestic macroeconomy via crisis contagion. But there may also be other channels at play. Rapid credit growth abroad may partially reflect additional foreign lending by domestic financial institutions, increasing the exposure of the domestic financial system to developments across borders. In addition, changes in global credit growth may reflect shifts in global sentiment and risk aversion, which could, in turn, affect the sentiment of domestic agents.[19]

Overall, these results indicate that, holding domestic factors fixed, financial developments abroad can significantly influence the conditional distribution of future GDP growth, with particularly large effects at the left tail. This points to an important role for cross-border spillovers in driving downside macroeconomic tail risk. It also suggests there is important information in foreign variables relevant for estimating GDP tail risk, over and above the information in domestic variables—a point we return to in the following sub-sections.

Robustness These headline results are robust to a range of alternative model specifications. We present results for a number of robustness exercises in Appendix B.2. These include: using bilateral (and time-invariant) financial weights to construct foreign variables using an alternative weighting scheme; replacing the FCI with an alternative measure of financial-market volatility; extending the foreign country set to include emerging market economies such that $N^* > N$; estimating the model on data prior to the GFC only (from 1981Q1 to 2005Q4); including a broader set of covariates in the domestic covariate set $\mathbf{x}_{i,t}$; and using a nested version of our model in which only US variables are included in the foreign variable set. Across specifications, we find that measures of foreign financial conditions and foreign credit growth are significantly associated with GDP-at-Risk, with effects that are larger at the left tail than at the median.

[19] Given that we control for domestic credit-to-GDP growth, we likely partial out some of these spillover effects via changes in sentiment. We consider a specification that accounts for contemporaneous spillovers from global to domestic credit growth in Section 4.

3.2.2 Cross-Country Heterogeneity

Because it is estimated on a panel of countries, our baseline model assumes coefficient homogeneity across countries. However, in principle, there may be differences across countries, both for domestic and foreign covariates, which could limit the use of the panel approach. Given our focus on the latter in this paper, we focus on potential differences in this sub-section, discussing two alternative ways to assess coefficient heterogeneity in our setup.

Openness As discussed above, our setup can be extended to allow for cross-country coefficient differences with respect to observable economic factors. Here, we assess whether they may be heterogeneity in the size of cross-country spillovers driven by differences in countries' levels of economic openness.[20] Returning to our notation from Section 2, we extend our baseline specification with interaction terms to account for cross-country heterogeneity in coefficients on foreign covariates as follows:

$$Q_{\Delta^h y_{i,t+h}}(\tau|\mathbf{x}_{i,t}, \mathbf{x}^*_{i,t}, open_i) = \alpha^h_i(\tau) + \mathbf{x}'_{i,t}\boldsymbol{\beta}^h(\tau) + \mathbf{x}^*_{i,t}{}'\boldsymbol{\vartheta}^h(\tau) + \left(open_i \cdot \mathbf{x}^*_{i,t}{}'\right)\boldsymbol{\delta}^h(\tau) \tag{5}$$

where $open_i$ is country-specific scalar capturing the (average) level of openness of country i over the sample.[21] The coefficient on the interaction term $\boldsymbol{\delta}^h(\tau)$, a $K \times 1$ vector, captures how the association between foreign variables and conditional quantiles of the GDP growth varies with a country's level of openness.

We estimate equation (5) using the same sample as for our baseline specification and include the same domestic and foreign covariates. In addition, we calculate each country's total trade-to-GDP and take its average over time, using this as our measure of openness $open_i$.[22]

The results—shown in Appendix B.4—indicate that the *ceteris paribus* association between foreign covariates and domestic GDP-a-Risk is larger for more open countries. However, estimates of the interaction term $\boldsymbol{\delta}^h(\tau)$ are generally insignificantly different from zero across variables and horizons. As a consequence, our results suggest that although there are some cross-country differences in the size of coefficients on foreign covariates associated with differences in countries' openness, such heterogeneity is not a significant feature for our sample.

Country-Specific Regressions We can also estimate equation (1) for a single country at a time. This yields 10 sets of estimated coefficients. Given that this procedure greatly reduces

[20] Our approach of using interaction terms to analyse the effect of openness on the size of cross-country spillovers within a panel setting mirrors that in, e.g., Cesa-Bianchi et al. (2019a) and Coman and Lloyd (2022).

[21] In this specification, because $open_i$ is not time-varying, we do not need to include it separately in regression (5), as it is absorbed in the fixed effect. Our headline findings in this section are robust the use of time-varying openness measures, for which we also include $open_{i,t}$ as a separate regressor.

[22] We also construct a measure of financial openness as the stock of each country's portfolio debt claims on non-residents as a percent of GDP. Results are qualitatively unchanged when using financial openness in place of trade openness.

the number of observations with which to robustly estimate coefficients, the error bands are substantially wider reflecting increased estimation uncertainty. In general, individual country coefficient estimates are statistically indistinguishable from the baseline panel estimation.[23] Nevertheless, we return to other properties of these country-specific regressions in our out-of-sample analysis in Section 3.3.2 to compare country-specific forecasting performance to our baseline panel specification, where there appear to be efficiency gains from pooling.

3.2.3 Model Fit

Our coefficient estimates highlight an important role for cross-border spillovers in driving downside macroeconomic tail risk. We now turn to a natural and complementary question: whether the inclusion of foreign covariates in equation (1) significantly improves estimates of the predicted conditional distribution of GDP growth. This question is of particular relevance when viewing the model as a tool for monitoring financial stability risks. Even if developments abroad are a key driver of downside risks to domestic GDP growth, there may be little gain—from a monitoring and forecasting perspective—to including them in the model if the information contained in foreign variables is already sufficiently captured in domestic indicators. In such a case, estimates of GDP-at-Risk would be little changed when accounting for foreign factors.

To formally test the additional explanatory power provided by foreign-weighted variables, we compute a horizon- and quantile-specific $R^1_h(\tau)$ statistic (or relative quantile score)—a goodness-of-fit measure for quantile regression analogous to the conventional R^2 statistic for OLS regression (Koenker and Machado, 1999). While the R^2 quantifies the success of one model relative to another—typically a constant-only model—at the conditional mean, the $R^1(\tau)$ provides information on the relative performance of models at the τ-th quantile. The $R^1_h(\tau)$ statistic is defined as: $R^1_h(\tau) = 1 - \frac{\hat{V}^h(\tau)}{\tilde{V}^h(\tau)}$, where $R^1_h(\tau) \in [0, 1]$, $\hat{V}^h(\tau)$ is the sum of weighted absolute residuals from the *unrestricted* foreign-augmented model at the τ-th quantile and h-th horizon, and $\tilde{V}^h(\tau)$ is the equivalent quantity from a *restricted* model.[24]

To focus on the additional information from foreign variables, we compare the full (unrestricted) foreign-augmented model to two restricted alternatives: a model with domestic quarterly GDP growth as the only covariate; and a domestic-only model with the domestic FCI, domestic credit-to-GDP growth and domestic GDP growth as explanatory variables. In the latter case, we interpret $R^1_h(\tau)$ as a measure of how much foreign augmentation alters the goodness-of-fit of the estimated τ-th quantile of h-quarter-ahead real GDP growth relative to

[23] As an additional robustness exercise, we use these country-specific estimates to compute the mean (and median) coefficient across countries—a quantile regression equivalent of the pooled mean-group estimator for linear regression (Pesaran, Shin, and Smith, 1999). The results, presented in Appendix B.5, indicate that the estimated pooled mean (and median) estimates are similar to those from the panel model.

[24] Formally, $V^h(\tau)$ is defined as: $V^h(\tau) = \sum_{t=1}^{T} \sum_{i=1}^{N} \rho_\tau \left(\hat{u}^h_{i,t}(\tau)\right)$ where $\rho_\tau \equiv \rho_\tau(u) = u\left[\tau - \mathbf{1}(u < 0)\right]$ is the check function and $\hat{u}^h_{i,t}(\tau) = \Delta^h y_{i,t+h} - \hat{Q}_{\Delta^h y_{i,t+h}}(\tau)$ are residuals from the quantile regression.

Table 2: In-sample $R_h^1(\tau)$ statistics for unrestricted foreign-augmented model across horizons and quantiles versus GDP-only and domestic-only restricted models

Horizons	Quantiles $\tau = 0.05$	$\tau = 0.25$	$\tau = 0.5$	$\tau = 0.75$	$\tau = 0.95$
	A: Foreign-Augmented (Unrestricted) vs. GDP-Only (Restricted)				
$h = 1$	0.19***	0.06*	0.03	0.03	0.05**
$h = 4$	0.19***	0.05	0.02	0.03	0.06***
$h = 8$	0.14***	0.06	0.03	0.05	0.07***
$h = 12$	0.14***	0.11***	0.08**	0.10	0.10***
$h = 20$	0.17***	0.23***	0.19***	0.19***	0.14***
	B: Foreign-Augmented (Unrestricted) vs. Domestic-Only (Restricted)				
$h = 1$	0.09***	0.03***	0.02*	0.02	0.04***
$h = 4$	0.07***	0.02**	0.02	0.02	0.05***
$h = 8$	0.07***	0.01	0.01	0.02	0.04***
$h = 12$	0.06***	0.04***	0.02	0.02	0.02***
$h = 20$	0.08***	0.13***	0.08***	0.04***	0.03***

Note: $R_h^1(\tau)$ statistics comparing the foreign-augmented (unrestricted) model to the domestic-GDP-only (Panel A) and domestic-only (Panel B) restricted models across horizons and quantiles. Significance at 10%, 5% and 1% levels denoted by *, ** and ***, respectively. Statistical significance assessed using likelihood-ratio test from Koenker and Machado (1999).

the domestic-only model, with a higher $R_h^1(\tau)$ denoting a larger increase in goodness-of-fit arising from the addition of foreign variables.

Table 2 details the in-sample $R_h^1(\tau)$ statistics across a range of quantiles and horizons for the whole panel of 10 countries, against the GDP-only specification in panel A and the domestic-only specification in panel B. As the restricted model is a nested version of the unrestricted model in both cases, we report the statistical significance of these statistics in Table 2 by applying the likelihood ratio test of Koenker and Machado (1999). Two observations are noteworthy. First, out to at least the 3-year horizon, the $R_h^1(\tau)$ is highest for the 5th percentile of real GDP growth, relative to both the median and the 95th percentile, in comparison to both the GDP-only and domestic-only restricted specifications. So, the inclusion of foreign variables in equation (1) improves estimates of the left tail of the conditional GDP-growth distribution most materially.

Second, while the in-sample $R_h^1(0.05)$ statistics peak in the near term (at $h = 1$), they remain around similar levels out to longer horizons (including $h = 20$). In sample, these values indicate the significant influence of medium-term drivers of estimated GDP-at-Risk. Therefore, the inclusion of foreign variables has significant explanatory power for the left tail of the real GDP-growth distribution across horizons. This is consistent with the coefficient estimates across quantiles presented in Figure 1.

Moreover, as we demonstrate in Table 3, these patterns hold for most countries within the sample—although consistent with the discussion in Section 3.2.2, the results do demonstrate that the importance of foreign variables differs somewhat across countries. Here, we present

Table 3: Country heterogeneity in in-sample $R_h^1(0.05)$ statistics for unrestricted foreign-augmented panel model across horizons versus GDP-only and domestic-only restricted models

	Countries									
	AUS	CAN	FRA	DEU	ITA	ESP	SWE	CHE	GBR	USA
	A: Foreign-Augmented (Unrestricted) vs. GDP-Only (Restricted)									
$h = 1$	0.14	0.21	0.08	0.28	0.25	0.31	0.22	-0.02	0.15	0.17
$h = 4$	0.14	0.27	0.15	0.16	0.14	0.23	0.17	0.19	0.18	0.27
$h = 8$	0.01	0.18	0.33	0.07	0.25	-0.10	0.19	0.05	0.20	0.23
$h = 12$	-0.17	0.15	0.35	0.09	0.07	0.04	0.18	0.19	0.19	0.32
$h = 20$	-0.48	0.17	0.36	0.02	0.35	0.25	0.02	0.21	0.15	0.28
	B: Foreign-Augmented (Unrestricted) vs. Domestic-Only (Restricted)									
$h = 1$	0.13	0.10	0.16	0.18	0.24	0.00	0.07	0.06	-0.10	0.02
$h = 4$	0.13	0.13	0.19	0.06	0.19	-0.12	0.02	0.13	-0.08	0.02
$h = 8$	-0.06	0.07	0.11	0.31	0.00	-0.14	0.04	0.15	0.07	0.05
$h = 12$	-0.12	0.04	0.16	0.31	-0.21	0.01	0.09	0.08	0.11	0.10
$h = 20$	-0.11	0.20	0.22	0.36	0.04	0.08	-0.18	-0.05	0.07	0.06

Note: $R_h^1(0.05)$ statistics comparing the foreign-augmented (unrestricted) panel model to the GDP-only (Panel A) and domestic-only (Panel B) restricted panel models across horizons and countries.

$R_h^1(0.05)$ statistics calculated at the country level across a range horizons.[25] The improvement in fit at the 5th percentile from the inclusion of foreign variables is largest for Germany and France, two of the most open countries in the euro area. In contrast, for some horizons, the $R_h^1(0.05)$ statistics are negative for Australia, indicating a worsening in fit from the inclusion of foreign variables. This could potentially be explained by the fact that Australia is relatively less open (the second least open country in our sample), and that some of its major trading partners are not included in the foreign country sample-set in our baseline specification.

We also assess the extent to which there are patterns over time in quantile-score metrics, which could signal when it pays off to consider international information to predict GDP-at-Risk. To do so, we calculate the average tick loss across countries at each point in time t, defined as:

$$TL_t^h(\tau) = \frac{1}{N}\sum_{i=1}^{N} \rho_\tau\left(\hat{u}_{i,t}^h(\tau)\right) \tag{6}$$

Note that the tick loss in turn determines $V_h(\tau)$, and so the $R_h^1(\tau)$ statistic. As such, it can be interpreted as a measure of goodness of fit over time.[26] We analyze the tick loss over time at the 5th percentile across different horizons, where a higher value for a specification implies a worse fit. More detail is provided in Appendix B.3. Across horizons, the improvement in fit for the foreign-augmented model can be seen most clearly around crisis episodes. For example, at the 1-quarter horizon, the gain from foreign-augmentation is seen most clearly around the

[25] Unlike in Table 2, we cannot report the formal significance of these statistics, as the comparison across models is not exactly nested—as evidenced by some negative $R_h^1(0.05)$ values.

[26] Giacomini and Komunjer (2005) show the tick loss to be suitable for evaluating quantile forecasts.

early-1990s recession and again around the GFC. This suggests that it pays off most to consider international information to predict GDP-at-Risk around extreme crisis episodes.

We additionally assess the extent to which our results are driven *entirely* by the period around the 2007-2008 GFC. To do so, we compute $R_h^1(\tau)$ statistics by excluding the GFC from the sample—specifically the period from 2006Q1 to 2009Q2. The headline results from Table 2 are robust to this. $R_h^1(\tau)$ statistics remain positive, at the 5th percentile especially. This indicates that the additional in-sample explanatory power attributable to foreign variables in our model, over and above domestic ones, is not solely driven by the GFC, although crises episodes are an important driver of the improved model fit from foreign augmentation.

3.2.4 Estimated Moments and Distributions

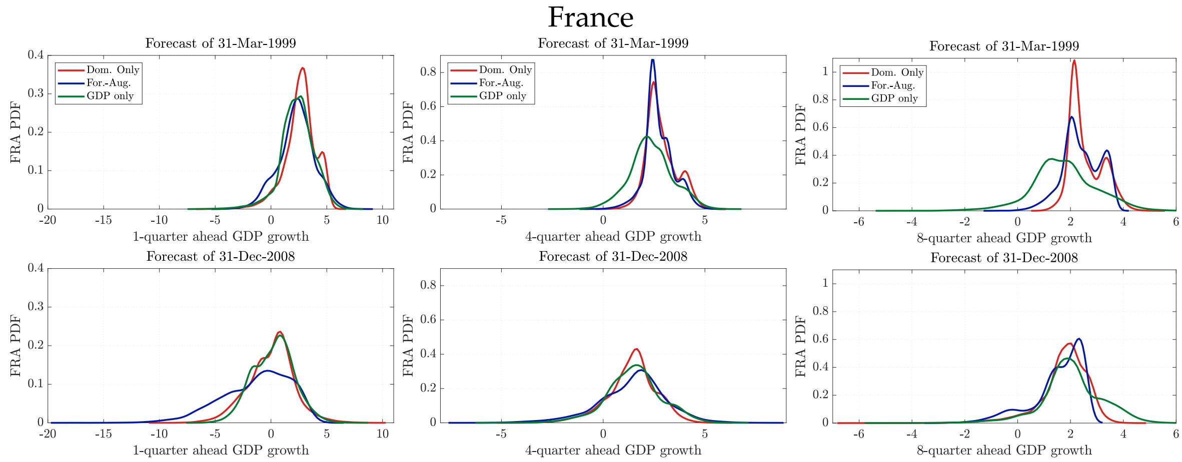
Building on these findings, we now investigate which *features* of the GDP-growth distribution the model can be informative about. We assess how in-sample estimates of GDP-growth moments and distributions from our foreign-augmented, domestic-only and domestic GDP-only models vary over the cycle, in particular in the run-up to crisis episodes.

Following Mitchell et al. (2021), we employ a non-parametric approach to recover an estimate of the full conditional density of GDP growth from the quantiles estimated using our quantile regression model. Unlike the approach in Adrian et al. (2019), who fit a skewed-t distribution to estimated quantiles, this method does not superimpose a specific functional form on the estimated quantiles to recover a predictive density. Instead, the conditional quantile estimates are mapped directly to a conditional density, assuming only local uniformity between the quantile forecasts. This non-parametric approach has the advantage that it does not rule out by construction certain features of the GDP-growth distribution that may be present in the data (e.g., multi-modality). To construct these densities, we estimate the quantile regression model at $\tau = 0.05, 0.10, 0.15, ..., 0.95$ and then smooth/interpolate across adjacent quantiles to approximate the true predictive density.[27]

For this sub-section, we focus on results for a single example country in the panel, France. We choose France as an example because (as discussed above) it stands out, along with Germany, as a country for which the improvement in fit at the 5th percentile from the inclusion of foreign variables is largest in Table 3, although we discuss the generality of our findings for other countries. Figure 2 presents in-sample predictive distributions for France for two points in time—1999Q1 and 2008Q4—fitted using the non-parametric approach described above. We choose these dates as examples because of the stark differences in macro-financial conditions at these times. The late 1990s were characterized by a period of relatively low credit growth

[27]For more details on this approach, see Algorithm 1 in Mitchell et al. (2021). Like them, we use a normal distribution to fit to extreme quantiles, i.e., below the 5th and above the 95th percentile. We choose to fit to these 19 quantiles specifically given evidence in Mitchell et al. (2021) that this is sufficient for accurate estimates of the true distribution. We avoid quantile crossing by rearranging quantiles estimated in the quantile regression as necessary following the approach of Chernozhukov, Fernandez-Val, and Galichon (2010).

Figure 2: Estimated in-sample non-parametric predictive densities



Note: Fitted probability density functions for GDP growth in France at the 1-, 4- and 8-quarter horizons. Densities constructed by fitting non-parametric density to quantile regression output in sample, following method of Mitchell et al. (2021). They represent predictions of GDP growth outturns in 1999Q1 and 2008Q4, i.e., formed in 1998Q4 and 2008Q3 (1-quarter-ahead), 1998Q1 and 2007Q4 (4-quarter) and 1997Q1 and 2006Q4 (8-quarter). Blue line shows estimates from the foreign-augmented model, red line shows estimates from the restricted domestic-only model, and green line shows estimates from a model with only domestic GDP growth.

(relative to GDP) across countries in our panel, as well as low volatility in financial markets, while the period preceding the GFC was characterized by high and rising credit growth, and by a sharp tightening in financial conditions at the height of the crisis. All the plots in Figure 2 compare the fitted distributions from the unrestricted foreign-augmented model to the two restricted, domestic-only and domestic GDP-only, models described above.

At the 1-quarter-ahead horizon (left panels), we compare densities formed in 2008Q3 (i.e., estimates of the GDP-growth distribution for 2008Q4), the quarter in which Lehman Brothers failed, relative to those formed in 1998Q4 (i.e., estimates of the 1999Q1 distribution). Predictive densities from the foreign-augmented model in 2008Q3 are not only further to the left (i.e., lower estimated mean), but are also flatter (i.e., higher estimated variance), more left-skewed, and fatter-tailed than densities formed in 1998Q4. We see similar interpretable moves in the estimated distributions at medium-term horizons too. In particular, the estimated GDP-growth distributions formed in 2007Q4 (i.e., 4 quarters prior to 2008Q4, middle panels) and 2006Q4 (8 quarters, right panels) are flatter and more-left skewed than corresponding estimated distributions in the late 1990s.

The differences between the estimated distributions from foreign-augmented, domestic-only and GDP-only models are also notable. At the 1-quarter horizon, especially, the estimated predictive density from the foreign-augmented model is much flatter and more left-skewed ahead of the GFC than for the domestic- and GDP-only models. This highlights that periods of tighter financial market conditions abroad are associated with a worsening in domestic

Table 4: Estimated correlation between mean and higher-order growth moments across countries

		AUS	CAN	FRA	DEU	ITA	ESP	SWE	CHE	GBR	USA
$h=1$	Variance	-0.8	-0.7	-0.8	-0.8	-0.8	-0.8	-0.8	-0.8	-0.9	-0.8
	Skew	0.5	0.5	0.5	0.5	0.5	0.3	0.5	0.4	0.5	0.3
$h=4$	Variance	-0.8	-0.6	-0.7	-0.7	-0.8	-0.8	-0.8	-0.8	-0.9	-0.9
	Skew	0.2	0.3	0.2	0.3	0.1	0.4	0.3	0.3	0.3	0.2
$h=8$	Variance	-0.7	-0.5	-0.7	-0.4	-0.8	-0.7	-0.8	-0.6	-0.9	-0.8
	Skew	0.2	-0.2	0.1	-0.1	0.4	0.0	0.1	0.0	0.4	0.4

Note: Estimates of the correlation over time between the mean and higher-order GDP-growth moments across countries. Moments are estimated by first estimating conditional quantiles of GDP-growth from the baseline (foreign-augmented) specification, and then by fitting a non-parametric distribution to these quantiles following Mitchell et al. (2021).

growth-at-risk, due to changes in higher moments of the domestic GDP-growth distribution. At medium-term horizons (at $h = 4$ and $h = 8$), the differences between the foreign-augmented model and the domestic- and GDP-only models are less stark. However, at the 8-quarter horizon specifically, the foreign-augmented model estimates a secondary recessionary mode by 2006Q4 not present in either the domestic-only or GDP-only estimates.[28] This is consistent with the fact that the rate of credit growth (relative to GDP) was not significantly above its historical average in France in the years preceding the GFC, and so it is only with the addition of foreign variables that the model picks up a pronounced rise in the probability of recession during these years.

These findings hold more generally across countries too. As shown in Appendix B.7, in the run-up to the GFC, across countries we find the estimated mean of GDP growth falls, while estimated variance rises and skew falls. These changes tend to be larger for the foreign-augmented model relative to the domestic-only specification.

Moreover, our foreign-augmented model provides estimates of higher-order GDP-growth moments that are interpretable over the business cycle, and in line with other studies. To demonstrate this, Table 4, shows that the conditional mean of the GDP-growth distribution is highly correlated with higher-order moments. In our foreign-augmented model, we find a correlation of around -0.8 between the 1-quarter-ahead mean and variance (i.e., counter-cyclical variance), and a correlation of around 0.5 between the 1-quarter-ahead mean and skewness (i.e., pro-cyclical skewness) on average across countries. These stylized facts have been been established in a range of other empirical literature (see, e.g., Adrian, Boyarchenko, and Giannone, 2019; Delle Monache, De Polis, and Petrella, 2021; Iseringhausen, Petrella, and Theodoridis, 2021), and so our findings lend support to the interpretability of estimated moments within our foreign-augmented model.

[28] Note that this multi-modality would be ruled out by construction by fitting skewed-t densities to estimated quantiles as in Adrian et al. (2019).

3.3 Out-of-Sample Estimation

We now turn to the real-time performance of our model and the extent to which foreign variables also provide additional explanatory power *out of sample*. To do so, we back-test the model by estimating it in real time from 1995Q1 onwards, with the caveats that we use final revised data only and do not account for delays or ragged-edges originating from the data release calendar. We estimate regression (1) across quantiles, extending the sample one quarter at a time from 1995Q1. This yields a 22-year quarterly time series of estimated coefficients and out-of-sample forecasts.

Although the magnitudes of corresponding real-time coefficient estimates vary somewhat over time as the sample is extended, they are robustly negative for foreign-weighted variables at each period. Moreover, for foreign-weighted credit-to-GDP especially, the coefficient estimate for the 5th percentile is consistently more negative than the median estimate, supporting our conclusion that foreign factors weigh on the left tail of domestic GDP growth in particular. These estimates, presented in Appendix B.8, provide an additional source of robustness analysis for our model.

Using these estimates, we assess two aspects of out-of-sample model performance, complementing the in-sample discussion in the previous sub-section. We first evaluate out-of-sample model fit and forecast accuracy, before discussing the narrative around estimated out-of-sample predictive moments and distributions.

3.3.1 Out-of-Sample Forecasting Accuracy

We assess the out-of-sample model fit and accuracy by analyzing the quantile scores and implied probability integral transforms (PIT) from the model. In line with previous studies into the out-of-sample performance of GDP-at-Risk models (e.g., Plagborg-Møller et al., 2020; Brownlees and Souza, 2021), we focus this analysis on near-term horizons (up to $h = 4$). Like existing work, we find that for longer horizons (beyond $h = 4$), the out-of-sample performance of all models is typically poor, and lower than for a simple GDP-only specification.

Quantile Scores We first follow the approach from Section 3.2.3 by computing out-of-sample quantile scores from our model compared to GDP-only and domestic-only restricted benchmarks. Table 5 presents the estimated $R^1_h(\tau)$ statistics up to $h = 4$, with statistical significance assessed using Diebold and Mariano (1995) statistics. As before, a positive $R^1_h(\tau)$ denotes an improvement in goodness-of-fit for the foreign-augmented model relative to the restricted model.

Panel A demonstrates that, at the 5th percentile, there is a sizeable, but predominantly insignificant, improvement in out-of-sample performance for the foreign-augmented model relative to the GDP-only model across near-term horizons. For other quantiles, there is lit-

Table 5: Out-of-sample $R_h^1(\tau)$ statistics for unrestricted foreign-augmented model at near-term horizons and across quantiles versus GDP-only and domestic-only restricted models

Horizons	$\tau = 0.05$	$\tau = 0.25$	$\tau = 0.5$	$\tau = 0.75$	$\tau = 0.95$
	A: Foreign-Augmented Panel vs. GDP-Only Panel				
$h = 1$	0.25*	0.06	0.02	-0.01	0.01
$h = 2$	0.20	0.03	0.00	-0.01	0.02
$h = 3$	0.17	0.01	-0.03	-0.04	0.03
$h = 4$	0.09	0.00	-0.05	-0.06	-0.04
	B: Foreign-Augmented Panel vs. Domestic-Only Panel				
$h = 1$	0.10	0.01	0.01	0.00	0.01
$h = 2$	0.06	0.00	0.00	0.00	0.03
$h = 3$	0.07	-0.01	-0.01	-0.03	0.06
$h = 4$	-0.04	-0.01	-0.03	-0.05	0.00
	C: Foreign-Augmented Panel vs. Country-Specific Foreign-Augmented				
$h = 1$	0.30***	0.13***	0.08***	0.12***	0.24***
$h = 2$	0.36***	0.15***	0.13***	0.17***	0.41***
$h = 3$	0.42***	0.19***	0.15***	0.18	0.44***
$h = 4$	0.46***	0.23***	0.17***	0.19	0.47***

Note: $R_h^1(\tau)$ statistics estimated out-of-sample comparing the foreign-augmented (unrestricted) model to the domestic-only (restricted) across near-term horizons and quantiles in panels A and B. Panel C compares out-of-sample comparison of foreign-augmented panel model with foreign-augmented country-specific model. Significance at 10%, 5% and 1% levels denoted by *, ** and ***, respectively. Statistical significance assessed using Diebold and Mariano (1995) test.

tle difference between the two models. Similarly, Panel B shows that the foreign-augmented model provides a considerable improvement in out-of-sample fit relative to the domestic-only model at the 5th percentile, and for horizons $h = 1$ to $h = 3$ specifically, albeit these improvements are statistically insignificant. Again, at other quantiles, there is little difference in out-of-sample performance between the two models.

As in Section 3.2.3, we also assess the average tick loss over time to assess when it pays off to incorporate foreign information to predict GDP-at-Risk. These are shown in Appendix B.9. Like for the in-sample analysis, the advantage of foreign-augmentation is most clearly visible around the GFC, particularly at $h = 1$.

Finally, we return to analyzing the country-specific regressions discussed in Section 3.2.2. While, in-sample, the country-specific model will trivially fit the data better than the panel model, this need not be the case out of sample. In Panel C, we report variants of the $R_h^1(\tau)$ statistics in which we compare the sum of weighted absolute residuals from the panel regression, in which coefficients are homogeneous across countries, to those from the country-specific regressions. Here, a higher value for $R_h^1(\tau)$ denotes a better out-of-sample fit for the panel model in comparison to the country-specific model. At all horizons, and at all quantiles, the statistic is positive and statistically significant, suggesting that there may be efficiency gains to using a panel model for out-of-sample analysis within our setup. Moreover, these efficiency benefits appear greatest at the left and right tails of the GDP distribution. So while it is possible to extend our baseline model to account for cross-country differences as we show in

Section 3.2.2, which will improve in-sample fit, doing so may not be costless for a practitioner by limiting available data to estimate coefficients reliably and detracting from out-of-sample performance.

Probability Integral Transform Complementary to our assessment of quantile scores, we also analyse the calibration of the predictive densities from our panel model by computing the empirical cumulative distribution of PITs.[29] This measures the percentage of observations that are below any given quantile. The closer the empirical cumulative distribution of the PITs is to the 45-degree line, the better calibrated the model is.[30] Results are presented in Appendix B.10 for the 1- and 4-quarter-ahead horizons with confidence bands constructed per the method of Rossi and Sekhposyan (2019).

The results illustrate that the foreign-augmented quantile regression generates robust predictive distributions. At both horizons, the empirical distribution of PITs for the foreign-augmented model is within the confidence bands at all quantiles for all countries (with the exception of Italy at the 1-quarter horizon). For this test, the improvement of fit of the foreign-augmented model is less visible relative to the domestic-only and GDP-only specifications.

3.3.2 Out-Of-Sample Estimated Moments and Distributions

We now turn to out-of-sample estimates of GDP-growth distributions, recreating predictive densities for 1999Q1 and 2008Q4 for France with real-time data. Using these estimates, we generally find weaker evidence of interpretable moves in higher moments of the GDP-growth distribution in the run-up to the GFC. [31]

Figure 3 presents out-of-sample estimates of predictive distributions for France in the run-up to the GFC. At the 1-quarter horizon, the results are similar to the in-sample estimates. The foreign-augmented model picks-up a notable flattening and worsening in downside-skew during the GFC compared to the prediction from the 1990s. And the addition of foreign variables leads to a substantial widening in the left-tail of the growth-distribution in particular. At the 4-quarter horizon, again the foreign-augmented model picks-up a rise in uncertainty and worsening in downside-skew in the run-up to the crisis. However the differences are less stark, and the estimates are similar to those from both the domestic-only and GDP-only models pointing to less clear evidence of additional information related to higher-order moments from foreign variables.[32]

[29]For this test, we recover estimates of the full forecast distribution at each point in time following the non-parametric approach described in Mitchell et al. (2021).

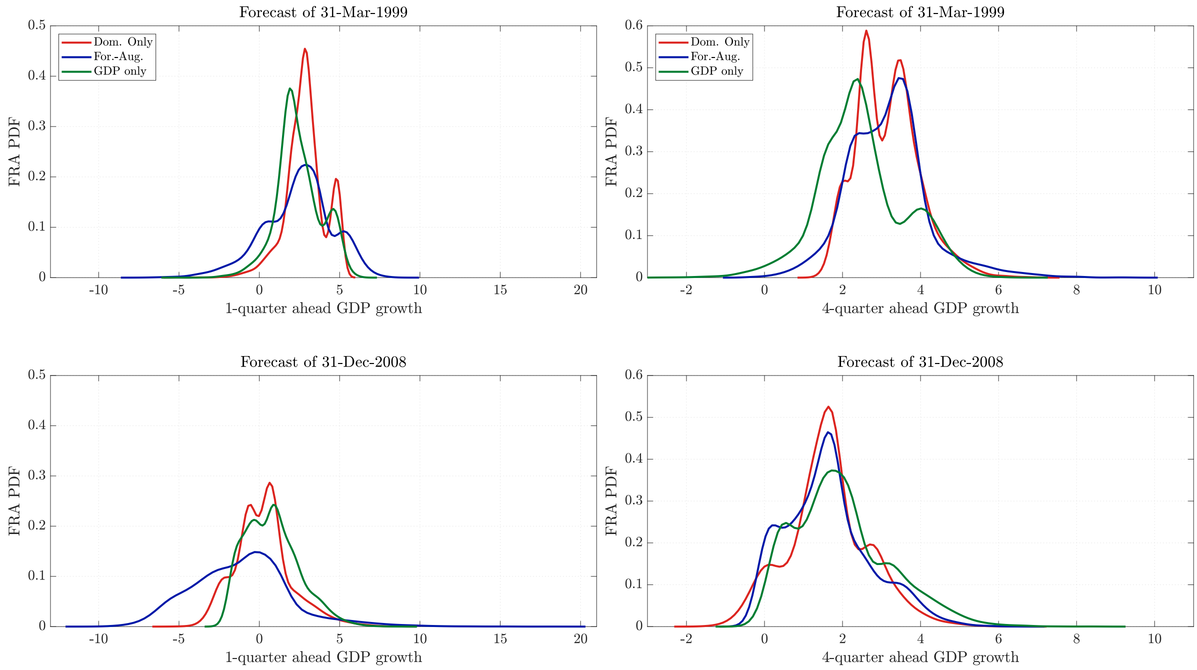
[30]In a perfectly calibrated model, the cumulative distribution of the PITs is a 45-degree line, so that the fraction of realisations below any given quantile $Q_{\Delta^h y_{i,t+h}}(\tau|\mathbf{X}_{i,t}, \mathbf{X}^*_{i,t})$ of the predictive distribution is exactly equal to τ.

[31]This is similar to findings in Plagborg-Møller et al. (2020). We similarly find weaker evidence of interpretability when looking across the entire panel of countries.

[32]We extend this analysis to other countries by assessing how estimated out-of-sample GDP-growth moments change across the panel in the run-up to the GFC. Similar to the results for France, we find some evidence of

Figure 3: Estimated out-of-sample non-parametric predictive densities

France



Note: Fitted probability density functions for France at the 1- and 4-quarter horizons. Densities constructed by fitting non-parametric density to quantile regression output out of sample, following method of Mitchell et al. (2021). They represent predictions of GDP growth outturns in 1999Q1 and 2008Q4, i.e., formed in 1998Q4 and 2008Q3 (1-quarter-ahead), 1998Q1 and 2007Q4 (4-quarter) and 1997Q1 and 2006Q4 (8-quarter). Blue line shows estimates from the foreign–augmented model, red line shows estimates from the restricted domestic-only model and green line shows estimates from a GDP-only model.

Overall, the results in this section have demonstrated that foreign-weighted variables exert a significant and robust influence on domestic GDP-at-Risk, even when accounting for domestic variables. Foreign-augmentation can provide additional information relevant for pinning down the left tail of the GDP-growth distribution in particular—in-sample this holds across horizons, but is limited to near-term horizons out of sample.

4 Structural Contribution of Foreign Drivers

So far, we have largely focused on the gains from accounting for foreign variables within a quantile-regression framework from a financial-stability monitoring perspective. In this section, we consider the importance of foreign developments for changes in the distribution of GDP in a more structural sense. As part of this, we seek to decompose historical estimates of GDP-at-Risk, assessing the contribution of foreign shocks to domestic tail risks. This mirrors

interpretable moves in higher-order moments (e.g., a rise in variance and fall in skew), albeit this evidence is weaker than the in-sample results (see Appendix B.11).

attempts to quantify the contribution of foreign shocks to domestic GDP growth at the mean (e.g., Cesa-Bianchi et al., 2019b), and offers a first assessment of how these factors might vary over the GDP distribution.

An immediate challenge to assessing the structural contribution of foreign variables to domestic GDP-at-Risk is the potential correlation of domestic and foreign covariates in equation (1). For example, consistent with evidence of a global financial cycle (Rey, 2013; Miranda-Agrippino and Rey, 2020), tighter financial conditions abroad could spill over to the domestic economy and generate a contemporaneous tightening in domestic financial conditions that, in turn, could drive changes in domestic GDP-at-Risk. The estimated coefficient on foreign financial conditions in equation (1) effectively partials out this effect however, by controlling for domestic financial conditions. So, simply decomposing the drivers of GDP-at-Risk using the fitted values from equation (1)—i.e., $\mathbf{x}'_{i,t}\hat{\boldsymbol{\beta}}^h(\tau)$ representing domestic drivers and $\mathbf{x}^{*\prime}_{i,t}\hat{\boldsymbol{\vartheta}}^h(\tau)$ foreign drivers—will likely not yield an accurate estimate for the relative importance of foreign shocks.

4.1 Method: Towards a Structural Decomposition

To move towards identifying the relative contribution of foreign and domestic *shocks* to domestic GDP-at-Risk, we build a decomposition using a two-step procedure.

In the first step, we orthogonalize the domestic variables with respect to their foreign-weighted counterparts. To do this, we estimate the following OLS regression for each domestic indicator $x^{(k)}_{i,t} \subset \mathbf{x}_{i,t}$ for $k = 1, ..., K$ and for each country $i = 1, ..., N$:

$$x^{(k)}_{i,t} = a^{(k)}_i + \mathbf{x}^{*}_{i,t}{}'\mathbf{b}^{(k)}_i + u^{(k)\perp}_{i,t} \tag{7}$$

where $a^{(k)}_i$ is a country- and indicator-specific scalar, while $\mathbf{b}^{(k)}_i$ is a $K \times 1$ vector and the scalar $u^{(k)\perp}_{i,t}$ represents the component of a domestic indicator $x^{(k)}_{i,t}$ that is orthogonal to contemporaneous variation in foreign-weighted indicators $\mathbf{X}^{*}_{i,t}$. Given coefficient estimates $\{\hat{a}^{(k)}_i, \hat{\mathbf{b}}^{(k)}_i\}$ from equation (7), we define the estimated orthogonal component as the residual: $\hat{u}^{(k)\perp}_{i,t} = x^{(k)}_{i,t} - \hat{a}^{(k)}_i - \mathbf{x}^{*}_{i,t}{}'\hat{\mathbf{b}}^{(k)}_i$.

In the second step, we then estimate a local-projection model for the conditional quantile function of h-period-ahead GDP growth using the estimated orthogonal component of domestic indicators, the full set of which is denoted by $\mathbf{u}^{\perp}_{i,t} = [\hat{u}^{(1)\perp}_{i,t}, ..., \hat{u}^{(K)\perp}_{i,t}]'$, alongside the set of weighted foreign variables $\mathbf{x}^{*}_{i,t}$:

$$Q_{\Delta^h y_{i,t+h}}(\tau|\mathbf{u}^{\perp}_{i,t}, \mathbf{x}^{*}_{i,t}) = \tilde{\alpha}^h_i(\tau) + \hat{\mathbf{u}}^{\perp\prime}_{i,t}\tilde{\boldsymbol{\beta}}^h(\tau) + \mathbf{x}^{*}_{i,t}{}'\tilde{\boldsymbol{\vartheta}}^h(\tau) \tag{8}$$

where we distinguish coefficients in this equation, relative to equation (1), with tildes. We can then decompose estimates of GDP-at-Risk by labelling $\hat{\mathbf{u}}^{\perp\prime}_{i,t}\tilde{\boldsymbol{\beta}}^h(\tau)$ as *domestic drivers* and

$\mathbf{x}_{i,t}^{*}{}'\tilde{\boldsymbol{\vartheta}}^{h}(\tau)$ as *foreign drivers*.

The key assumption in this procedure is that foreign indicators can contemporaneously influence domestic ones, but domestic indicators cannot contemporaneously affect their foreign counterparts. In effect, we treat the domestic country as a small-open economy, by excluding instantaneous feedback from domestic variables to foreign ones. This mirrors the block exogeneity assumption that has been widely used to estimate the transmission of shocks at the mean using structural vector autoregression methods in the empirical international macroeconomics literature (e.g., Dedola, Rivolta, and Stracca, 2017; Cesa-Bianchi and Sokol, 2022). Moreover, building on Cesa-Bianchi et al. (2019b), we show in Appendix C.2 how this two-step procedure is observationally equivalent, up to a scalar, to estimating a factor model to identify the effects of global developments.

However, there are caveats to the block-exogeneity assumption. As such, we interpret our results with some caution. First, to the extent that the assumption precludes within-quarter transmission from domestic economies to the rest of the world, we view estimates as an upper bound for the contribution of foreign shocks to domestic macroeconomic tail risks.

Second, while this procedure does orthogonalize foreign-weighted variables with respect to their domestic counterparts, it does not enable a structural decomposition of shocks *within* countries. So, the approach can isolate the relative importance of foreign or global shocks for domestic GDP-at-Risk. But it cannot distinguish between, for example, different types of structural shock within that (e.g., shocks to financial conditions vs. credit-growth shocks).

4.2 Results: Contribution of Foreign Shocks to GDP-at-Risk

We apply this orthogonalization procedure to our baseline empirical model to estimate the relative importance of foreign drivers of GDP-at-Risk. To do so, we make one change to the baseline model outlined in Section 3.1. To justify the small-open economy assumption implicit in the orthogonalization, we exclude the US from the set of domestic economies when estimating the structural decompositions. Nevertheless, we include the US in the foreign variable set, so we continue to account for its influence in the global economy.

We present coefficient estimates from regression (8) for our baseline specification in Appendix C.1. These complement our earlier estimates in Figure 1. Unlike Figure 1, these estimates capture contemporaneous spillovers of global factors to domestic covariates—and as such, global variables have a larger impact on domestic tail risk. In Appendix C.3, we also present orthogonalized decompositions for the estimated 5th percentile of 3-year-ahead real GDP growth. These decompositions indicate the differing importance of foreign and domestic shocks for GDP-at-Risk over time.

We also use this orthogonalized model to assess the relative importance of foreign shocks in driving tail risk more systematically. To do so, we estimate the model using the two-step pro-

Table 6: Share of Variation in Fitted Values (%) Attributed to Foreign Shocks Across Horizons

	$h = 1$		$h = 4$		$h = 12$	
Country	$\tau = 0.05$	0.5	0.05	0.5	0.05	0.5
AUS	97.73	86.42	91.73	80.29	57.86	61.38
CAN	98.39	87.60	89.03	79.26	42.69	46.86
FRA	97.39	91.20	90.47	83.27	43.77	46.84
DEU	97.19	86.63	89.76	79.78	51.66	53.34
ITA	97.43	91.76	95.35	89.16	63.01	68.38
ESP	97.74	87.30	91.86	80.88	54.34	59.92
SWE	97.03	87.49	94.37	85.17	65.71	67.68
CHE	98.71	91.60	91.61	85.60	45.01	45.93
GBR	98.68	89.28	95.67	87.82	76.65	80.30
Avg.	*97.81*	*88.81*	*92.21*	*83.47*	*55.63*	*58.96*

Note: Share of variation at the 5th percentile ($\tau = 0.05$) and median ($\tau = 0.5$) of country-GDP distributions at different horizons: $h = 1$ (1 quarter), $h = 4$ (1 year), and $h = 12$ (3 years). Share definition in equation (9). Shares constructed from baseline model in which domestic indicators are orthogonalized with respect to all foreign indicators, akin to a small-open economy assumption for domestic countries.

cedure described above, and investigate variation in each countries' GDP-at-Risk in turn. For each country i, equation (7) imposes $\text{cov}_t(\mathbf{x}^*_{i,t}, \hat{\mathbf{u}}^{\perp}_{i,t}) = \mathbf{0}$, and so the variance of the time series of fitted values of the τ-th percentile of the GDP-growth distribution in country i, $\Delta^h \hat{y}_{i,t+h}(\tau)$, can be decomposed as:

$$\text{var}_t\left(\Delta^h \hat{y}_{i,t+h}(\tau)\right) = \text{var}_t\left(\hat{\mathbf{u}}^{\perp\prime}_{i,t}\hat{\hat{\boldsymbol{\beta}}}^h(\tau)\right) + \text{var}_t\left(\mathbf{x}^{*\prime}_{i,t}\hat{\hat{\boldsymbol{\vartheta}}}^h(\tau)\right)$$

for each i. With these variances constructed from the time series of estimates, then the share of variation in the fitted value of country-i GDP growth at the τ-th percentile at horizon h attributable to foreign sources can be defined as:

$$ForShare^h_i(\tau) \equiv 100 \times \left[\frac{\widehat{\text{var}}_t\left(\mathbf{x}^{*\prime}_{i,t}\hat{\hat{\boldsymbol{\vartheta}}}^h(\tau)\right)}{\widehat{\text{var}}_t\left(\Delta^h \hat{y}_{i,t+h}(\tau)\right)}\right] \tag{9}$$

The estimated shares $ForShare^h_i(\tau)$ at $h = 1, 4, 12$ and $\tau = 0.05$ for each country in our baseline regression are presented in Table 6. As discussed, we interpret these quantities as upper-bound estimates for the share of variation attributable to foreign sources due to the stringency of the orthogonalization assumption imposed by equation (7).

Overall, Table 6 illustrates that a substantial portion of variation in estimates of the GDP-growth distribution can be attributed to foreign sources for all 9 countries in our sample. At the 1-quarter horizon, the average share of variation in GDP-at-Risk (i.e., $\tau = 0.05$) from foreign sources is 98%, around 9pp more than the variation attributed to foreign sources at the median ($\tau = 0.5$). While the share of variation attributable to foreign sources tends to decline as the horizon increases, the relative importance of foreign factors remains substantial. At the 1-year horizon, the average share of GDP-at-Risk variation linked with foreign shocks is 92% at the 5th percentile, around 9pp more than at the median. These findings emphasize the crucial role for foreign vulnerabilities at the left tail of the GDP growth distribution specifically. It suggests that there may be important cross-border contagion effects in the global economy that amplify the macroeconomic consequences of tail events over and above more general interdependence between nations (Forbes, 2012).

Table 6 also demonstrates that there is variation across countries. The UK, unsurprisingly given its position as a small-open international centre, stands out as the country most exposed to foreign shocks. Around 77% of variation in the left-tail of its GDP-growth distribution over the 3-year horizon is associated with foreign shocks, 21pp more than the cross-country average.

Robustness Finally, we assess the robustness of these results, present the details in Appendix C.3. First, we estimate our baseline model, but construct foreign-weighted variables using bilateral financial weights from BIS International Banking Statistics. These estimates again suggest that the role of foreign shocks is especially important at the left-tail of the distribution with the share of variation in the left-tail of the GDP-growth distribution 3pp, 11pp and 11pp greater than the corresponding estimates at the median at the 1-, 4- and 12-quarter horizons, respectively. Second, we estimate a model with more domestic covariates, mirroring the specification in Aikman et al. (2019). We continue to exclude the US from the domestic variable set when constructing these decompositions, but include them in the domestic covariate set. This is an important robustness test for our purposes, as the inclusion of additional domestic covariates makes it more challenging for foreign shocks to explain a substantial proportion of variation in estimated tail risk. Despite that, we continue to find that the share of variation in the estimated 5th percentile of GDP growth attributable to foreign shocks is 89%, 80% and 39% at the 1-, 4- and 12-quarter horizons, respectively. So overall, our key finding—that foreign factors play a substantial role in explaining variation in the estimated 5th percentile of GDP growth, and often more so than for the median—is robust across model specifications.

5 Conclusion

This paper has shown that foreign vulnerabilities matter for domestic macroeconomic tail risks. Faster global credit-to-GDP growth and tighter global financial conditions exert a significant negative influence on the left tail of the GDP-growth distribution. Moreover, these foreign indicators provide information relevant for estimating domestic GDP-at-Risk, over and above domestic ones, both in and out of sample. Decomposing historical estimates of GDP-at-Risk into orthogonalized domestic and foreign shocks, we show that foreign vulnerabilities on average explain up to around 90% of variation over the 1-year horizon, more than the comparable figure for the median.

Taken together, our findings have important implications for macroprudential policymakers. By highlighting the relevance of global spillovers, they emphasize the importance of monitoring global variables when assessing risks to domestic financial stability. Moreover, by demonstrating the substantial contribution of foreign shocks to domestic tail risks, they point to the potential benefits of international macroprudential policy cooperation in response to global shocks. Additionally, our general methodology can be applied more widely, for instance to inform analyses of GDP-at-Risk within emerging-market economies, where assessments of tail risks have been more limited in spite of their substantial exposures to foreign events.

References

ADRIAN, T., N. BOYARCHENKO, AND D. GIANNONE (2019): "Vulnerable Growth," *American Economic Review*, 109, 1263–1289.

ADRIAN, T., F. GRINBERG, N. LIANG, S. MALIK, AND J. YU (2022): "The Term Structure of Growth-at-Risk," *American Economic Journal: Macroeconomics*, Forthcoming.

AIKMAN, D., J. BRIDGES, S. BURGESS, R. GALLETLY, I. LEVINA, C. O'NEILL, AND A. VARADI (2018): "Measuring risks to UK financial stability," Bank of England working papers 738, Bank of England.

AIKMAN, D., J. BRIDGES, S. HACIOGLU HOKE, C. O'NEILL, AND A. RAJA (2019): "Credit, capital and crises: a GDP-at-Risk approach," Bank of England working papers 824, Bank of England.

BARRO, R. J. (2009): "Rare Disasters, Asset Prices, and Welfare Costs," *American Economic Review*, 99, 243–264.

BARRO, R. J. AND J. F. URSÚA (2012): "Rare Macroeconomic Disasters," *Annual Review of Economics*, 4, 83–109.

BLUWSTEIN, K., M. BUCKMANN, A. JOSEPH, M. KANG, S. KAPADIA, AND O. SIMSEK (2020): "Credit growth, the yield curve and financial crisis prediction: evidence from a machine learning approach," Bank of England working papers 848, Bank of England.

BROWNLEES, C. AND A. B. SOUZA (2021): "Backtesting global Growth-at-Risk," *Journal of Monetary Economics*, 118, 312–330.

BUSETTI, F., M. CAIVANO, D. DELLE MONACHE, AND C. PACELLA (2021): "The time-varying risk of Italian GDP," *Economic Modelling*, 101.

CANAY, I. A. (2011): "A simple approach to quantile regression for panel data," *The Econometrics Journal*, 14, 368–386.

CARNEY, M. (2020): "The Grand Unifying Theory (and practice) of Macroprudential Policy," Speech at Logan Hall, University College London, Bank of England.

CESA-BIANCHI, A., R. DICKINSON, S. KÖSEM, S. LLOYD, AND E. MANUEL (2021): "No economy is an island: how foreign shocks affect UK macrofinancial stability," *Bank of England Quarterly Bulletin*, 61.

CESA-BIANCHI, A., F. EGUREN-MARTIN, AND G. THWAITES (2019a): "Foreign booms, domestic busts: The global dimension of banking crises," *Journal of Financial Intermediation*, 37, 58–74.

CESA-BIANCHI, A., M. H. PESARAN, AND A. REBUCCI (2019b): "Uncertainty and Economic Activity: A Multicountry Perspective," *The Review of Financial Studies*, 33, 3393–3445.

CESA-BIANCHI, A. AND A. SOKOL (2022): "Financial shocks, credit spreads and the international credit channel," *Journal of International Economics*, 135.

CHAVLEISHVILI, S. AND S. MANGANELLI (2019): "Forecasting and stress testing with quantile vector autoregression," Working Paper Series 2330, European Central Bank.

CHERNOZHUKOV, V., I. FERNANDEZ-VAL, AND A. GALICHON (2010): "Quantile and Probability Curves without Crossing," Sciences Po publications info:hdl:2441/5rkqqmvrn4t, Sciences Po.

CLARK, T. E., F. HUBER, G. KOOP, M. MARCELLINO, AND M. PFARRHOFER (2021): "Investigating Growth at Risk Using a Multi-country Non-parametric Quantile Factor Model," Papers 2110.03411, arXiv.org.

COMAN, A. AND S. P. LLOYD (2022): "In the face of spillovers: Prudential policies in emerging economies," *Journal of International Money and Finance*, 122.

CORSETTI, G. (2008): *New Open Economy Macroeconomics*, London: Palgrave Macmillan UK, 1–10.

DEBARSY, N., C. ERTUR, AND J. P. LESAGE (2012): "Interpreting dynamic space–time panel data models," *Statistical Methodology*, 9, 158–171, special Issue on Astrostatistics + Special Issue on Spatial Statistics.

DEDOLA, L., G. RIVOLTA, AND L. STRACCA (2017): "If the Fed sneezes, who catches a cold?" *Journal of International Economics*, 108, S23–S41.

DELLE MONACHE, D., A. DE POLIS, AND I. PETRELLA (2021): "Modeling and forecasting macroeconomic downside risk," Temi di discussione (Economic working papers) 1324, Bank of Italy, Economic Research and International Relations Area.

DIEBOLD, F. AND R. MARIANO (1995): "Comparing Predictive Accuracy," *Journal of Business and Economic Statistics*, 13, 253–63.

DUPREY, T. AND A. UEBERFELDT (2020): "Managing GDP Tail Risk," Staff Working Papers 20-3, Bank of Canada.

EGUREN-MARTIN, F. AND A. SOKOL (2019): "Look abroad! Global financial conditions and risks to domestic growth," Bank underground blog, Bank of England.

EICKMEIER, S. AND T. NG (2015): "How do US credit supply shocks propagate internationally? A GVAR approach," *European Economic Review*, 74, 128–145.

FARHI, E. AND X. GABAIX (2016): "Rare Disasters and Exchange Rates," *The Quarterly Journal of Economics*, 131, 1–52.

FERRARA, L., M. MOGLIANI, AND J.-G. SAHUC (2022): "High-frequency monitoring of growth at risk," *International Journal of Forecasting*, 38, 582–595.

FORBES, K. (2012): "The 'Big C': Identifying Contagion," NBER Working Papers 18465, National Bureau of Economic Research, Inc.

FRANTA, M. AND L. GAMBACORTA (2020): "On the effects of macroprudential policies on Growth-at-Risk," *Economics Letters*, 196.

GABAIX, X. (2012): "Variable Rare Disasters: An Exactly Solved Framework for Ten Puzzles in Macro-Finance," *The Quarterly Journal of Economics*, 127, 645–700.

GALÁN, J. E. (2020): "The benefits are at the tail: Uncovering the impact of macroprudential policy on growth-at-risk," *Journal of Financial Stability*, 100831.

GALVAO, A. F., J. GU, AND S. VOLGUSHEV (2020): "On the unbiased asymptotic normality of quantile regression with fixed effects," *Journal of Econometrics*, 218, 178–215.

GALVAO, A. F. AND G. MONTES-ROJAS (2015): "On Bootstrap Inference for Quantile Regression Panel Data: A Monte Carlo Study," *Econometrics*, 3, 1–13.

GIACOMINI, R. AND I. KOMUNJER (2005): "Evaluation and Combination of Conditional Quantile Forecasts," *Journal of Business & Economic Statistics*, 23, 416–431.

GIGLIO, S., B. KELLY, AND S. PRUITT (2016): "Systemic risk and the macroeconomy: An empirical evaluation," *Journal of Financial Economics*, 119, 457–471.

GOURIO, F. (2012): "Disaster Risk and Business Cycles," *American Economic Review*, 102, 2734–2766.

GOURIO, F., M. SIEMER, AND A. VERDELHAN (2013): "International risk cycles," *Journal of International Economics*, 89, 471–484.

ISERINGHAUSEN, M., I. PETRELLA, AND K. THEODORIDIS (2021): "Aggregate Skewness and the Business Cycle," Cardiff Economics Working Papers E2021/30, Cardiff University, Cardiff Business School, Economics Section.

KAPETANIOS, G. (2008): "A bootstrap procedure for panel data sets with many cross-sectional units," *The Econometrics Journal*, 377–395.

KATO, K., A. F. GALVAO, AND G. V. MONTES-ROJAS (2012): "Asymptotics for panel quantile regression models with individual effects," *Journal of Econometrics*, 170, 76–91.

KOENKER, R. W. AND G. BASSETT (1978): "Regression Quantiles," *Econometrica*, 46, 33–50.

KOENKER, R. W. AND J. A. F. MACHADO (1999): "Goodness of Fit and Related Inference Processes for Quantile Regression," *Journal of the American Statistical Association*, 94, 1296–1310.

KOOP, G. AND D. KOROBILIS (2014): "A new index of financial conditions," *European Economic Review*, 71, 101–116.

LAMARCHE, C. (2021): "Quantile Regression for Panel Data and Factor Models," *Oxford Research Encyclopedia of Economics and Finance*.

LHUSSIER, S. (2022): "Financial conditions and macroeconomic downside risks in the euro area," *European Economic Review*, 143.

LO DUCA, M. AND T. A. PELTONEN (2013): "Assessing systemic risks and predicting systemic events," *Journal of Banking and Finance*, 37, 2183–2195.

LORIA, F., C. MATTHES, AND D. ZHANG (2019): "Assessing Macroeconomic Tail Risk," Finance and Economics Discussion Series 2019-026, Board of Governors of the Federal Reserve System (U.S.).

MIRANDA-AGRIPPINO, S. AND H. REY (2020): "U.S. Monetary Policy and the Global Financial Cycle," *The Review of Economic Studies*, 87, 2754–2776.

MITCHELL, J., A. POON, AND D. ZHU (2021): "Multimodality in Macroeconomic Dynamics: Constructing Density Forecasts from Quantile Regressions," Working paper, Federal Reserve Bank of Cleveland.

PESARAN, M. H., T. SCHUERMANN, AND S. M. WEINER (2004): "Modeling Regional Interdependencies Using a Global Error-Correcting Macroeconometric Model," *Journal of Business & Economic Statistics*, 22, 129–162.

PESARAN, M. H., Y. SHIN, AND R. P. SMITH (1999): "Pooled Mean Group Estimation of Dynamic Heterogeneous Panels," *Journal of the American Statistical Association*, 94, 621–634.

PLAGBORG-MØLLER, M., L. REICHLIN, G. RICCO, AND T. HASENZAGL (2020): "When Is Growth at Risk?" *Brookings Papers on Economic Activity*, 167–229.

REY, H. (2013): "Dilemma not trilemma: the global cycle and monetary policy independence," *Proceedings - Economic Policy Symposium - Jackson Hole*, 1–2.

ROSSI, B. AND T. SEKHPOSYAN (2019): "Alternative tests for correct specification of conditional predictive densities," *Journal of Econometrics*, 208, 638–657.

SCHULARICK, M. AND A. M. TAYLOR (2012): "Credit Booms Gone Bust: Monetary Policy, Leverage Cycles, and Financial Crises, 1870-2008," *American Economic Review*, 102, 1029–1061.

WACHTER, J. A. (2013): "Can Time-Varying Risk of Rare Disasters Explain Aggregate Stock Market Volatility?" *Journal of Finance*, 68, 987–1035.

WU, Y. AND Y. LIU (2009): "Variable Selection in Quantile Regression," *Statistica Sinica*, 19, 801–817.

Appendix

A Data Sources

Table 7 presents a full list of data sources used in this paper—both in the main body and the appendices.

Table 7: List of Data Sources

Variable	Source	Frequency	Notes
Dependent Variable			
Real GDP	OECD	Quarterly	Construct annual average growth across quarterly horizons
Covariates			
FCI	IMF	Quarterly	See (Adrian et al., 2022) and Koop and Korobilis (2014)
Equity Volatility	Datastream	Daily	Calculate realized volatility within quarter using standard deviation of daily returns
Credit-to-GDP	BIS	Quarterly	Construct 3-year change in ratio
House Prices	OECD	Quarterly	Construct 3-year growth
Capital Ratio	Aikman et al. (2019)	Annual	Ratio of tangible common equity to tangible assets
Inflation	OECD	Quarterly	Annual growth of CPI
Policy Rates	BIS	Quarterly	Annual change in central bank policy rates
Bilateral Weights			
Export Weights	IMF DOTS	Quarterly	Construct weights by averaging across each calendar year to smooth seasonal variation
Financial Weights	BIS IBS, Tbl. 9D	Quarterly	Construct weights by averaging across each calendar year to smooth seasonal variation

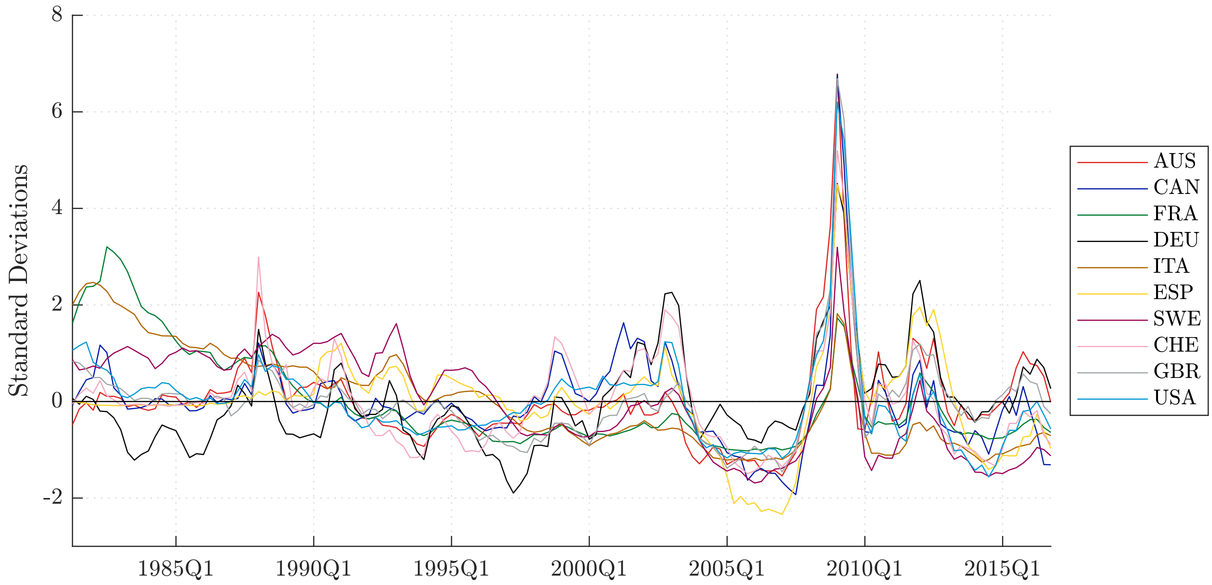
Table 8 presents the bivariate correlations between domestic and foreign-weighted FCI and credit-to-GDP for each country over the sample of the baseline specification (1981Q1-2016Q3). To support this, in Figures 4 and 5, we plot country-level time series of the two main explanatory variables used in our baseline specification: the FCI and the 3-year change in credit-to-GDP. The figures depict the standardised values of the series.

Table 8: Correlation Between Domestic Indicator and Foreign-Weighted Indicator for Each Country

	AUS	CAN	FRA	DEU	ITA	ESP	SWE	CHE	GBR	USA
FCI	0.834	0.899	0.567	0.477	0.563	0.837	0.740	0.872	0.916	0.935
Credit-to-GDP	0.716	-0.239	0.178	-0.265	0.757	0.634	0.746	-0.210	0.900	0.772

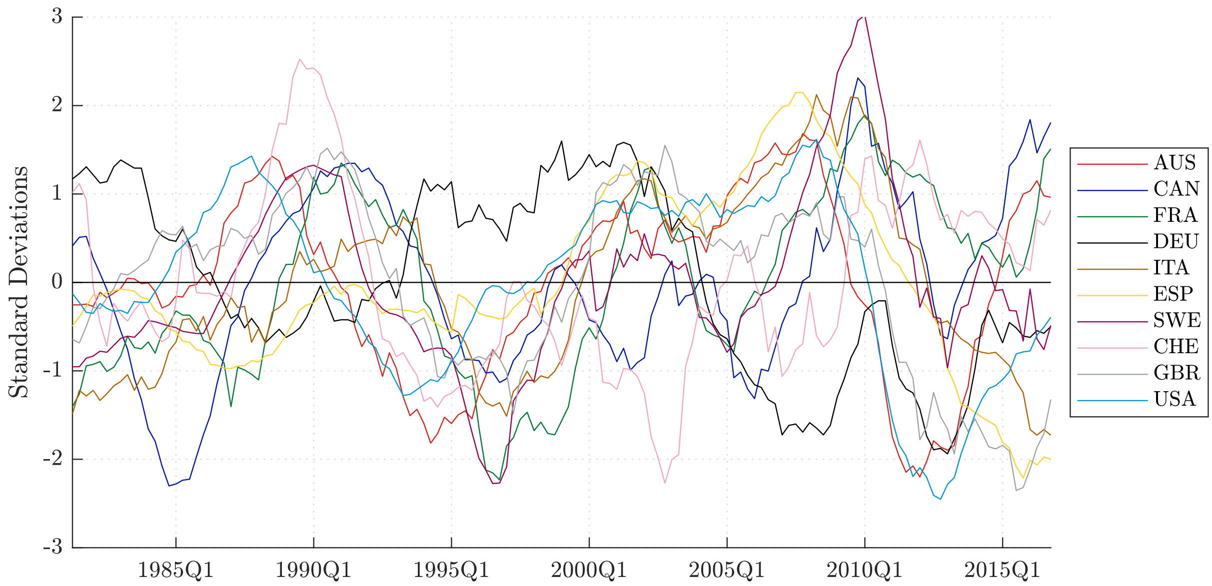
Note: Correlation between domestic indicator and corresponding foreign-weighted indicator for each country.

Figure 4: Country-Level Time Series of FCI used in Baseline Specification



Note: Standardized country-specific time series of FCI used in baseline specification over the period 1981Q1:2016Q3.

Figure 5: Country-Level Time Series of 3-year change in Credit-to-GDP used in Baseline Specification



Note: Standardized country-specific time series of 3-year change in credit-to-GDP used in baseline specification over the period 1981Q1:2016Q3.

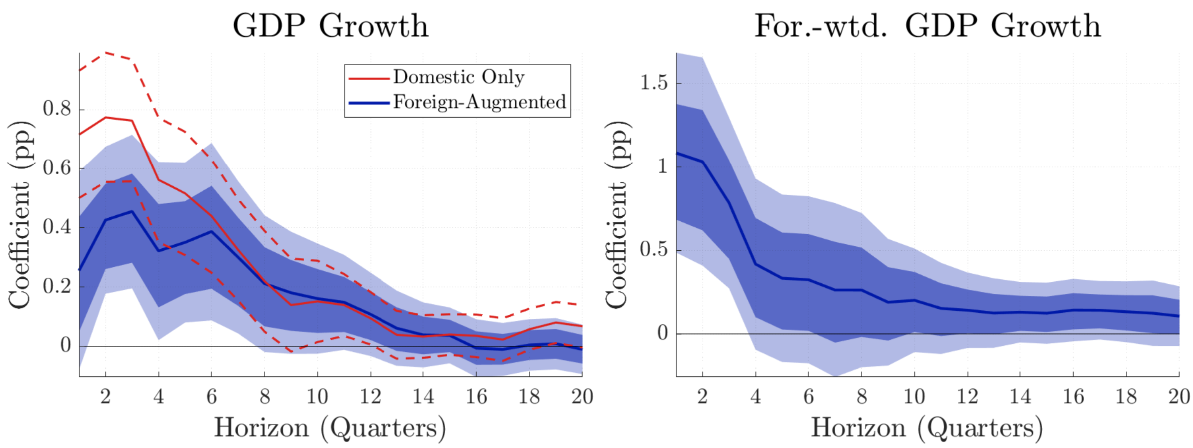
B Additional Results and Robustness Analysis

B.1 Additional Results from the Baseline Empirical Model

In this Appendix sub-section, we report additional results from our baseline empirical model described in Section 3.1.

Coefficient Estimates for Macroeconomic Controls Across Horizons Figure 6 presents coefficient estimates for the macroeconomic control variables—domestic and foreign-weighted quarterly real GDP growth—at the $\tau = 0.05$th quantile across horizons in our specific model described in Section 3. Both domestic and foreign-weighted real GDP growth are associated with higher estimates of the 5th percentile of real GDP growth.

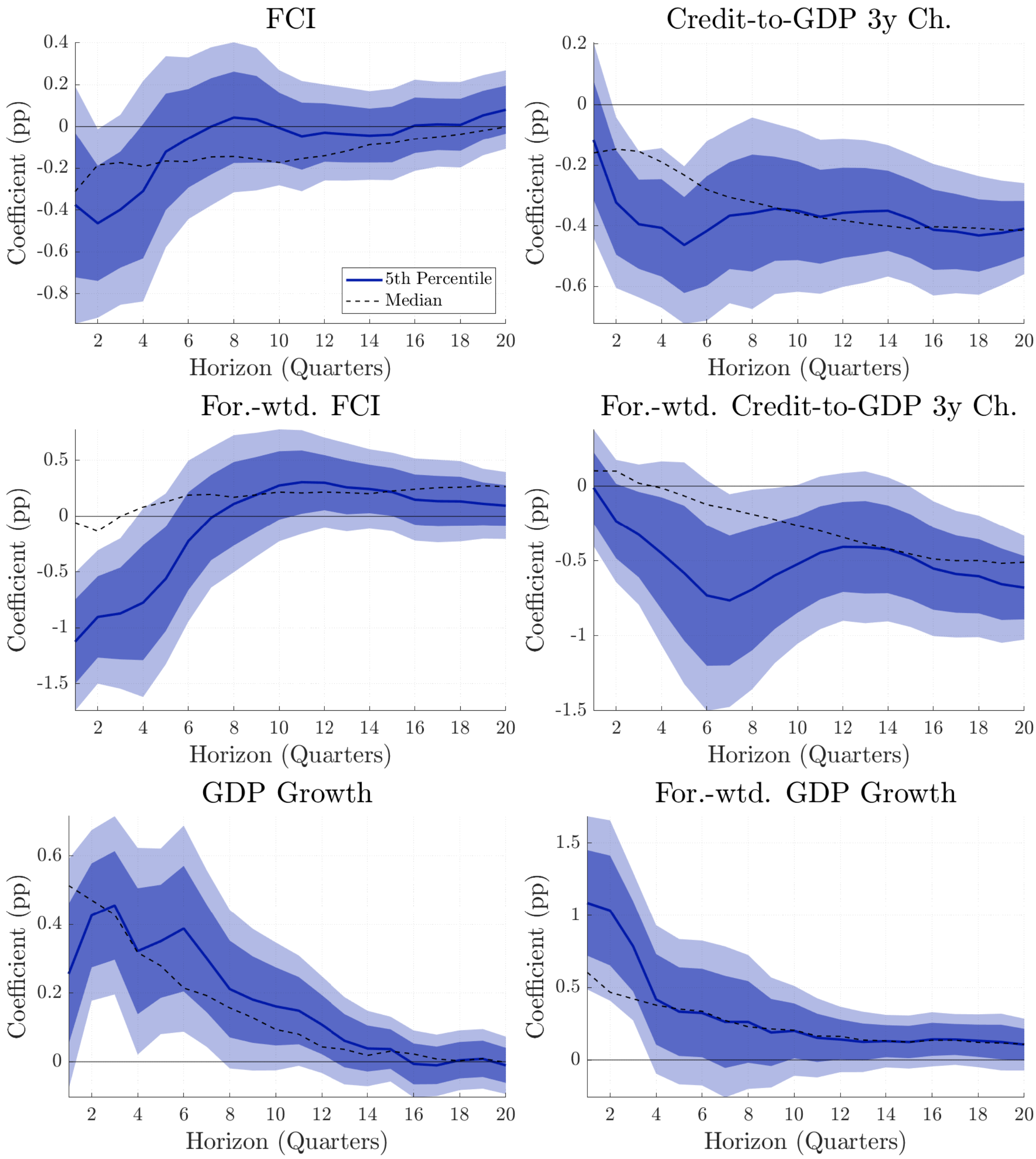
Figure 6: Association between indicators and the 5th percentile of GDP growth across horizons



Note: Estimated association between one standard deviation change in each indicator at time t with 5th percentile of average annual real GDP growth at each quarterly horizon. Red dashed lines denote 90% coefficient bands from the domestic-only model, that excludes foreign covariates. Solid blue lines denote coefficient estimates from model that includes foreign covariates. Light (dark) blue-shaded areas represent 90% (68%) confidence bands from block bootstrap procedure.

Coefficient Estimates Across Quantiles Figure 7 compares coefficient estimates from our baseline foreign-augmented model at the 5th and 50th percentiles. The coefficient estimates for FCI and credit highlight notable differences over the distribution. The near-term impacts of tighter financial conditions are more negative at the 5th percentile than at the median, while the inter-temporal reversal is also specific to the left tail. For foreign-weighted credit, coefficient estimates are negative at all horizons for the 5th percentile. But faster foreign credit growth is associated with higher median GDP growth in the near-term.

Figure 7: Association between indicators and GDP growth across horizons at the 5th and 50th percentiles



Note: Solid blue lines denote estimated association between one standard deviation change in each indicator at time t with 5th percentile of average annual real GDP growth at each quarterly horizon. Light (dark) blue-shaded areas represent 90% (68%) confidence bands around these estimates from block bootstrap procedure. Black dashed lines denote corresponding coefficient estimates at the 50th percentile (i.e., median).

B.2 Robustness: Coefficient Estimates

In this Appendix sub-section, we report key robustness exercises around the coefficient estimates in our baseline specification. Table 9 summarizes the results from these exercises, illustrating that our headline results are robust to a range of alternative model specifications. Across all specifications, the estimated coefficient on foreign-weighted credit-to-GDP at the 5th percentile for medium-term horizons is significantly negative at the 5% level at least. These coefficients, and those on the foreign FCI, are typically more negative than coefficients estimates for the median, indicating the influence of these foreign variables on the left tail of the conditional GDP growth distribution in particular. We discuss each of the robustness exercises in more detail below.

Foreign-Weighting Scheme In our baseline specification, we use trade weights to capture countries' bilateral exposures. These weights have the advantage of running back to 1980, enabling us to use time-varying weights. However, we find similar results when we use bilateral financial weights using BIS International Banking Statistics that capture banks' exposures to the rest of the world (Panel A).[33]

Alternate Financial Conditions Panel B presents results from a specification using a measure of within-quarter realized equity market volatility, used by Aikman et al. (2019), as an alternative to the FCI used in the baseline. This equity-market volatility series is available for 13 countries, relative to 10 in our baseline panel, and it extends to 2018Q4, relative to 2016Q3 in our baseline sample.

Foreign Country Set In Panel C, we present results from a specification where we extend the set of countries used to define foreign-weighted covariates. We increase our foreign country set (N^*) to 16, by including 6 emerging market economies (China, Korea, Indonesia, Mexico, Turkey and Hong Kong) in addition to the 10 advanced economies used in our baseline specification. We maintain our domestic variable set (N) at 10. We shorten the sample for this specification due to limited data availability in some emerging market economies. The results from this model are very similar to our baseline results, although we find slightly larger effects of foreign variables on domestic GDP-at-Risk when we extend the foreign country set.

Pre-GFC Sample To assess the extent to which the GFC drives our results, Panel D reports coefficients from a sample estimated on pre-GFC data, from 1981Q1 to 2005Q4. As in the full sample, we find that foreign-weighted vulnerabilities exert a significant influence on the left tail of GDP growth, with foreign credit weighing more on the 5th percentile in the medium

[33] Owing to data limitations, we construct time-invariant bilateral financial weights using average values from 2005 to 2018.

Table 9: Coefficient estimates from robustness exercises

	(A) Financial Weights					(B) Alternative Financial Conditions				
	$h=1$	$h=4$	$h=8$	$h=12$	$h=20$	$h=1$	$h=4$	$h=8$	$h=12$	$h=20$
Domestic Variables										
FCI/VIX	-0.740**	-0.360^	0.023	-0.052	0.048	0.096	0.103	0.105	0.066	-0.019
	[-0.284**]	[-0.157^]	[-0.121]	[-0.129]	[0.002]	[0.124]	[0.178^]	[0.008]	[0.003]	[-0.041]
Credit-to-GDP	-0.214^	-0.539***	-0.599**	-0.443**	-0.461***	-0.166^	-0.499***	-0.490***	-0.389***	-0.336***
	[-0.218*]	[-0.283**]	[-0.427***]	[-0.507***]	[-0.483***]	[-0.252*]	[-0.454***]	[-0.518***]	[-0.469***]	[-0.404***]
GDP growth	0.227	0.217^	0.149	0.097	-0.021	0.188^	0.258*	0.123^	0.088^	-0.033^
	[0.469***]	[0.287***]	[0.107^]	[-0.007]	[-0.032]	[0.274^]	[0.266***]	[0.131*]	[0.072^]	[-0.013]
Foreign Variables										
For. FCI/VIX	-0.886**	-0.904*	0.082	0.365^	0.122	0.722^	0.251^	-0.135	-0.160^	-0.017
	[-0.108]	[0.006]	[0.099]	[0.141]	[0.274^]	[0.440^]	[0.138]	[-0.029]	[-0.074]	[-0.023]
For. Credit-to-GDP	0.018	-0.222	-0.499^	-0.322	-0.650***	-0.243^	-0.597^	-0.655**	-0.429*	-0.341^
	[0.215*]	[0.161]	[0.065]	[-0.099]	[-0.476**]	[-0.066]	[-0.188^]	[-0.357^]	[-0.371^]	[-0.363^]
For. GDP growth	1.117***	0.646**	0.425^	0.263*	0.177^	1.171***	0.720**	0.201^	0.026	0.073^
	[0.746***]	[0.451***]	[0.302**]	[0.217**]	[0.142^]	[0.958***]	[0.635***]	[0.304*]	[0.111^]	[0.083^]
N (N^*)	10 (10)					13 (13)				
Weights (Sample)	Financial, Average (1981Q1-2016Q3)					Trade, Time-Varying (1981Q1-2018Q4)				

	(C) Extended Foreign Country Set					(D) Pre-GFC Sample				
	$h=1$	$h=4$	$h=8$	$h=12$	$h=20$	$h=1$	$h=4$	$h=8$	$h=12$	$h=20$
Domestic Variables										
FCI	-0.529^	-0.348	0.066	-0.048	0.217^	-0.523^	-0.296	-0.209^	-0.124	-0.014
	[-0.223^]	[-0.112]	[-0.066]	[-0.055]	[0.174^]	[-0.337**]	[-0.223*]	[-0.173^]	[-0.198*]	[-0.054]
Credit-to-GDP	0.033	-0.240^	-0.189^	-0.186^	-0.362***	0.016	-0.450*	-0.627***	-0.506***	-0.306***
	[0.031]	[0.004]	[-0.101]	[-0.244**]	[-0.327***]	[-0.221^]	[-0.287^]	[-0.318*]	[-0.352**]	[-0.365***]
GDP growth	0.553**	0.209^	0.106	0.134^	0.031	0.124	0.174^	-0.027	-0.043	-0.047*
	[0.592***]	[0.269**]	[0.095]	[0.060]	[0.059]	[0.311***]	[0.139^]	[0.076]	[-0.027]	[-0.051^]
Foreign Variables										
For. FCI	-1.550**	-1.271**	0.017	0.323^	0.013	-0.793*	0.186	0.349	0.153	-0.075
	[-0.442*]	[-0.276]	[-0.136]	[0.046]	[0.137]	[0.108]	[0.204]	[0.261^]	[0.225^]	[0.152]
For. Credit-to-GDP	0.095	-0.538^	-0.841**	-0.566**	-0.668***	0.165	-0.182	-0.421^	-0.441**	-0.392***
	[0.101]	[-0.071]	[-0.223]	[-0.353^]	[-0.470**]	[0.042]	[-0.163]	[-0.323*]	[-0.364**]	[-0.462***]
For. GDP growth	0.236	0.143	0.026	0.051	0.092^	0.806***	0.316*	0.180^	0.145*	0.035
	[0.248*]	[0.120^]	[0.004]	[0.028]	[0.056]	[0.603***]	[0.356***]	[0.190*]	[0.159**]	[0.087^]
N (N^*)	10 (16)					10 (10)				
Weights (Sample)	Trade, Time-Varying (1991Q3-2016Q3)					Trade, Time-Varying (1991Q3-2005Q4)				

	(E) Additional Domestic Covariates					(F) US-Only Foreign Variable Set				
	$h=1$	$h=4$	$h=8$	$h=12$	$h=20$	$h=1$	$h=4$	$h=8$	$h=12$	$h=20$
Domestic Variables										
FCI	-0.387^	-0.327	-0.059	-0.145	0.045	-0.703*	-0.687^	-0.237	-0.172^	-0.060
	[-0.279^]	[-0.209^]	[-0.170^]	[-0.158^]	[0.014]	[-0.476***]	[-0.344**]	[-0.255*]	[-0.164^]	[-0.034]
Credit-to-GDP	-0.133	-0.486***	-0.427***	-0.385***	-0.389***	-0.199	-0.567**	-0.652**	-0.604**	-0.481***
	[-0.248**]	[-0.312**]	[-0.391***]	[-0.440***]	[-0.343***]	[-0.269**]	[-0.399***]	[-0.510***]	[-0.549***]	[-0.438***]
GDP growth	0.384*	0.282^	0.145	0.050	-0.055	0.460*	0.427*	0.202^	0.078	0.037
	[0.491***]	[0.246**]	[0.114^]	[0.044]	[0.003]	[0.538***]	[0.300***]	[0.057]	[-0.017]	[-0.004]
House price growth	0.381*	0.297^	0.156	-0.036	-0.133^					
	[0.046]	[0.023]	[0.004]	[-0.065]	[-0.190**]					
Capital ratio	-0.230	-0.061	0.159	0.114	0.176^					
	[-0.028]	[-0.126]	[-0.149]	[-0.120]	[-0.027]					
Inflation	-0.634**	-0.418	-0.322	-0.047	0.047					
	[-0.249]	[-0.315^]	[-0.224]	[-0.148]	[-0.052]					
Policy Rate	-0.226	-0.531**	-0.569**	-0.396**	-0.168*					
	[-0.295**]	[-0.384***]	[-0.346***]	[-0.293**]	[-0.132*]					
Foreign Variables										
For. FCI	-0.742*	-0.449	0.200	0.282^	0.042	-0.968*	-0.426	0.509^	0.504*	0.217
	[-0.043]	[0.063]	[0.136]	[0.152]	[0.189^]	[0.187]	[0.279^]	[0.294^]	[0.201]	[0.313^]
For. Credit-to-GDP	-0.305	-0.464^	-0.405	-0.262	-0.609***	0.365^	-0.064	-0.292	-0.176	-0.617**
	[0.140^]	[0.033]	[-0.095]	[-0.272]	[-0.407**]	[0.309*]	[0.326^]	[0.292^]	[0.151]	[-0.543**]
For. GDP growth	1.108***	0.651*	0.399^	0.172^	0.065	0.669*	0.438^	0.485**	0.279**	0.154^
	[0.766***]	[0.484***]	[0.239**]	[0.120^]	[0.080]	[0.516***]	[0.356***]	[0.316***]	[0.260**]	[0.186**]
N (N^*)	10 (10)					10 (1)				
Weights (Sample)	Trade, Time-Varying (1981Q1-2016Q3)					US-Only Foreign Variable Set (1981Q1-2016Q3)				

Notes: Coefficient estimates for 5th pctile. [and median]. Significance, form block bootstrap, at 32%, 10%, 5% and 1% levels denoted by ^, *, ** and ***.

term than the median out to $h = 12$. In addition, pre-GFC coefficient estimates for foreign-weighted credit-to-GDP, in particular, are similar in magnitude to full-sample estimates at most horizons, suggesting that the GFC period is not driving our results.

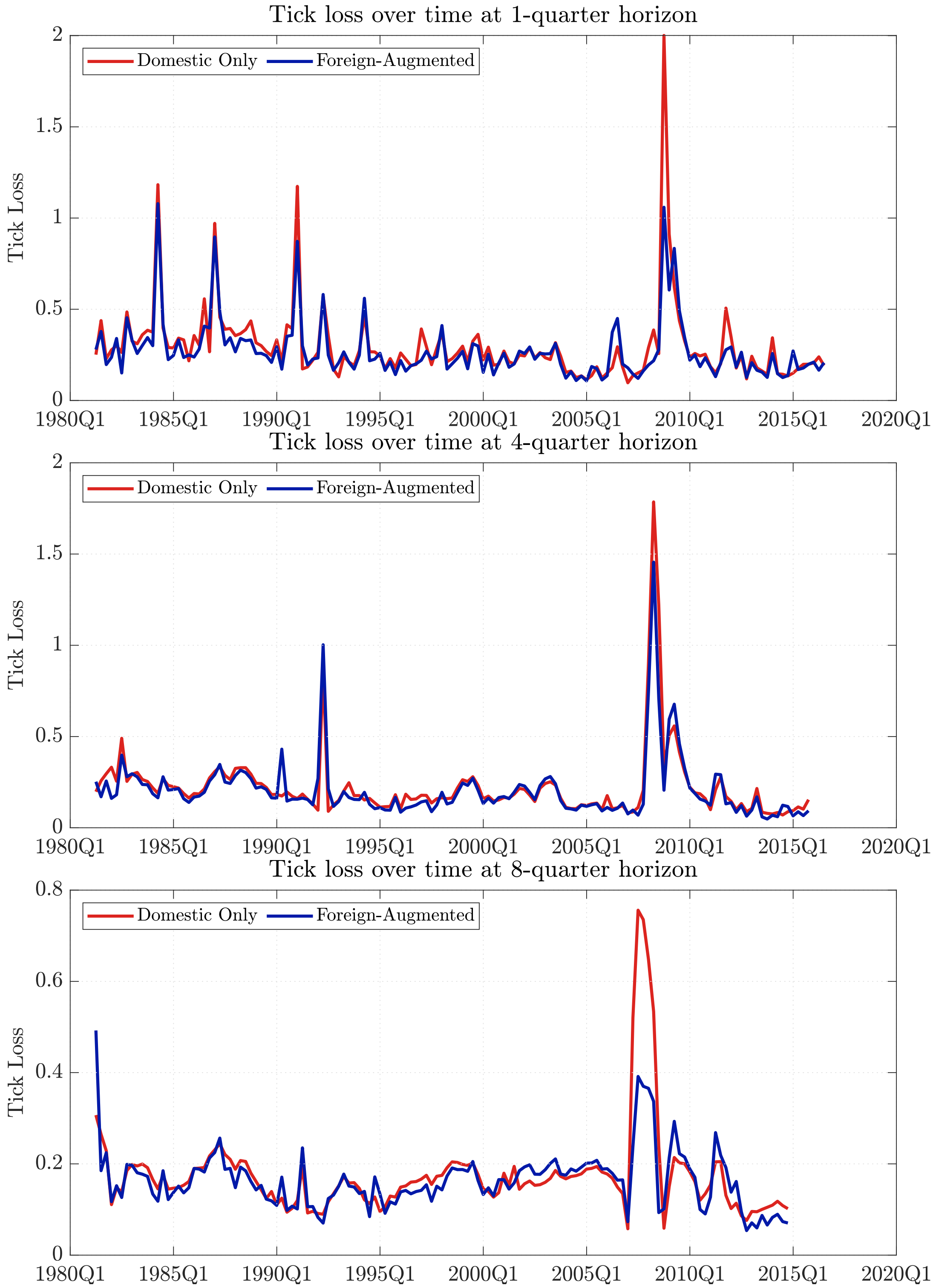
Domestic Covariates Panel E presents results from a specification with additional domestic covariates. Here, we include domestic 3-year house price growth, the capital ratio (a measure of overall banking system resilience), the 1-year change in headline central bank policy rates and 1-year inflation in our domestic covariate set—as in Aikman et al. (2019). This allows us to test whether foreign variables provide additional explanatory power, even after accounting for a much wider range of potential domestic covariates. Despite the addition of more domestic covariates, estimated coefficients on the foreign variables continue to remain significant and of similar magnitude to the baseline.

US-Only Foreign Variables In Section 2, we noted that our proposal nests one in which only US variables are used in the foreign variable set. When estimating this, we continue to find similar results: US financial market volatility weighs on domestic GDP-at-Risk in the near term, while US credit-to-GDP growth has a significant association with medium-term tails risks (Panel F). However, the magnitude of estimated coefficients is somewhat smaller. For example, at $h = 8$, the coefficient on US credit-to-GDP growth is -0.292 (insignificant), versus -0.691 (significant at the 10% level) in the baseline. While this indicates that the US plays an important role in driving domestic tail risks, there are advantages to using a wider set of countries when constructing the foreign-weighted variables to account for a broader set of cross-border transmission channels and shocks—including the build-up of regional risks.

B.3 Model Fit Over Time

Figure 8 presents estimates of the average tick loss at the 5th percentile across countries over time for both the domestic-only and foreign-augmented model at various horizons. This gives an indication of when exactly incorporating foreign information into the model serves to improve estimation of GDP-at-Risk. A lower tick loss implies an improvement in fit—and so time periods where the blue foreign-augmented line lies below the red domestic-only line highlight occasions when foreign information improves the estimation of GDP-at-Risk. In general, these charts highlight the pay-off from including foreign information is highest around crisis episodes, consistent with a focus on the 5th percentile of GDP growth. For example, at the 1-quarter horizon, the blue line lies clearly below the red line around the early 1990s—a period of recession for the majority of advanced economies in our panel—and again around the GFC. The improvement in fit around the GFC is also visible for the 4- and 8-quarter horizon.

Figure 8: Estimated tick loss at 5th percentile ($TL_t^h(0.05)$) over time



Note: Estimated average tick loss at the 5th percentile across countries over time and across horizons. The red line denotes estimates from the domestic-only specification and the blue line estimates from the foreign-augmented model.

B.4 Country Heterogeneity: Coefficient Estimates from Interaction Regression

Table 10 presents key results for a specification that includes an interaction term between foreign variables and a country's level of openness. Panel A shows estimated coefficients at the 5th percentile for the foreign variables and interaction terms, as well as their statistical significance. The interaction terms are generally insignificant across variables and horizons, only occasionally significant at the 32% level at most (e.g., the interaction term on foreign-weighted FCI at the 1-quarter horizon). This suggests that heterogeneity in country spillovers driven by differences in levels of observed openness is not a significant feature of our data.

To aid the economic interpretation of these results, Panel B shows the association between each foreign variable and GDP-at-Risk for different levels of openness. The first three rows correspond to the estimated association for a country with an average level of trade-to-GDP (around 54% in our sample), while the bottom three rows correspond to the estimated association for a country with level of trade-to-GDP one standard deviation above the average (around 74%). This demonstrates that, for example, at peak (at $h = 1$), a one-standard deviation rise in a country's openness increases the effect of foreign FCI on GDP-at-Risk by around a third (from -1.2pp to -1.6pp for a one standard deviation rise in foreign-weighted FCI). Similarly, at peak ($h = 8$), a one standard deviation rise in a country's openness increases the effect of foreign credit on GDP-at-risk by around a quarter (from -0.6pp to -0.8pp for a one standard deviation rise in foreign-weighted credit).

Table 10: Interaction Coefficient Estimates at 5th percentile from Interaction Regression (5)

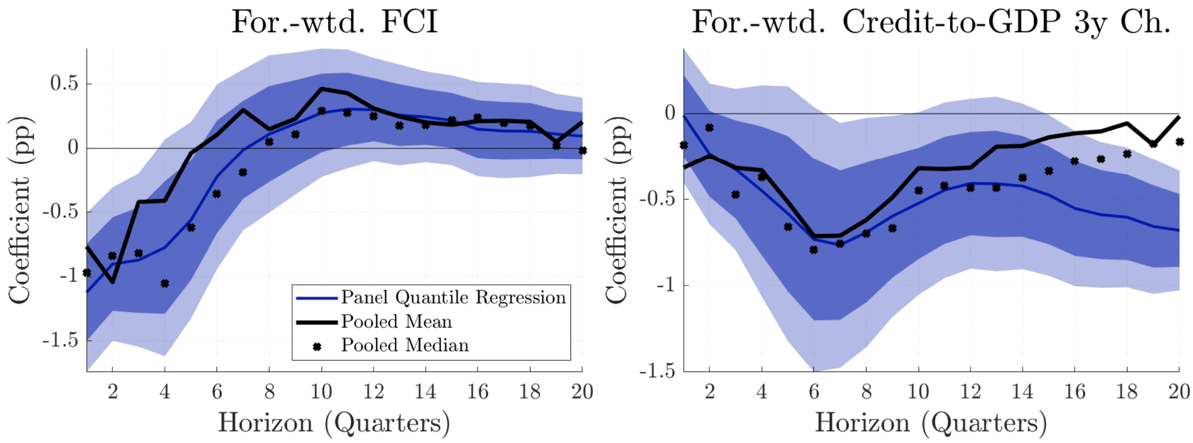
	Horizons				
	h =1	h =4	h =8	h =12	h =20
	A: Coefficient Estimates and Statistical Significance				
Foreign Variables					
For. Credit-to-GDP	-0.669^	-0.134	-0.197	-0.028	-1.137**
For. FCI	-0.191	-1.069^	-0.074	0.206	0.393^
For. GDP growth	1.071^	0.216	0.395	0.231	0.398*
Interaction Terms					
$open_i\times$ For. Credit-to-GDP	1.237	-0.514	-0.821	-0.716	0.731^
$open_i\times$ For. FCI	-1.962^	0.612	0.458	0.285	-0.619^
$open_i\times$ For. GDP growth	-0.043	0.396	-0.18	-0.135	-0.485^
	B: Economic Significance				
Average openness					
For. Credit-to-GDP	0.008	-0.405	-0.633	-0.403	-0.735
For. FCI	-1.248	-0.730	0.177	0.360	0.064
For. GDP growth	1.049	0.430	0.298	0.161	0.139
Average openness + 1 s.d.					
For. Credit-to-GDP	0.252	-0.506	-0.795	-0.543	-0.591
For. FCI	-1.635	-0.609	0.268	0.416	-0.057
For. GDP growth	1.042	0.508	0.262	0.135	0.044

Note: Panel A shows coefficient estimates for $\boldsymbol{\vartheta}^h(\tau)$ and $\boldsymbol{\delta}^h(\tau)$ from equation (5) at the 5th percentile, with significance from block bootstrap procedure at 32%, 10%, 5% and 1% levels denoted by ^, *, ** and ***, respectively. Panel B shows estimated association between a one standard deviation rise in each foreign variable and GDP-at-Risk for differing levels of trade openness.

B.5 Country Heterogeneity: Pooled Country-Specific Results

Figure 9 plots a comparison of our baseline coefficient estimates for foreign-weighted FCI and credit-to-GDP growth from a panel model with the mean and median of coefficient estimates from individual country regressions. The results indicate that the estimated pooled mean and median estimates are similar to those from the panel model.

Figure 9: Average association between indicators and the 5th percentile of GDP growth across horizons from country-specific regressions



Note: Estimated association between one standard deviation change in each indicator at time t with 5th percentile of annual average real GDP growth at each quarterly horizon. Black line denotes mean coefficient estimate when pooling individual-country estimates. Black crosses represent the median. Blue line denotes point estimates from our baseline panel model. Light (dark) blue-shaded areas represent 90% (68%) confidence bands from block bootstrap procedure for the corresponding coefficient estimate over the full 1981Q1-2018Q4 sample.

B.6 Robustness: In-Sample Model Fit

Table 11 reports $R_h^1(\tau)$ statistics for two of the robustness exercises, namely: calculating the statistics from the baseline foreign-augmented model, versus a restricted domestic-only model, using only the fitted values outside of the 2006-2008 period to ensure the GFC is not the sole driver of results; and assessing the fit of a foreign-augmented model with additional domestic covariates (as per Aikman et al., 2019), versus a restricted model with only these domestic variables. In both cases, we find that the addition of foreign-weighted covariates continues to result in a substantial increase in fit, especially at the 5th percentile, as in the baseline presented in the main body of the paper.

Table 11: $R_h^1(\tau)$ across horizons and quantiles for robustness exercises

	Quantiles				
Horizons	$\tau = 0.05$	$\tau = 0.25$	$\tau = 0.5$	$\tau = 0.75$	$\tau = 0.95$
	A: Baseline Foreign-Augmented vs. Domestic-Only, Excluding GFC				
$h = 1$	0.08	0.02	0.02	0.03	0.03
$h = 4$	0.05	0.02	0.02	0.03	0.06
$h = 8$	0.01	-0.01	0.00	0.02	0.04
$h = 12$	0.06	-0.02	-0.01	0.01	0.01
$h = 20$	0.08	0.08	0.03	0.02	0.02
	B: For.-Aug. with Additional Dom. Covariates vs. Extended Dom.-Only				
$h = 1$	0.10	0.05	0.03	0.04	0.04
$h = 4$	0.07	0.04	0.04	0.04	0.08
$h = 8$	0.04	0.02	0.02	0.02	0.06
$h = 12$	0.03	0.03	0.01	0.02	0.02
$h = 20$	0.03	0.08	0.05	0.03	0.02

Note: $R_h^1(\tau)$ statistics comparing the foreign-augmented (unrestricted) model to the domestic-only (restricted) across horizons and quantiles.

B.7 Time Variation in In-Sample Moments

Figures 10 and 11 present estimated changes in in-sample moments for each country prior to the GFC at the 1- and 4-quarter horizon, respectively.

B.8 Out-of-Sample Coefficient Stability

Figure 12 plots real-time coefficient estimates for domestic and foreign-weighted variables in the foreign-augmented model.

B.9 Out-of-Sample Model Fit Over Time

Figure 13 plots out-of-sample estimates of the average tick loss at the 5th percentile across countries over time.

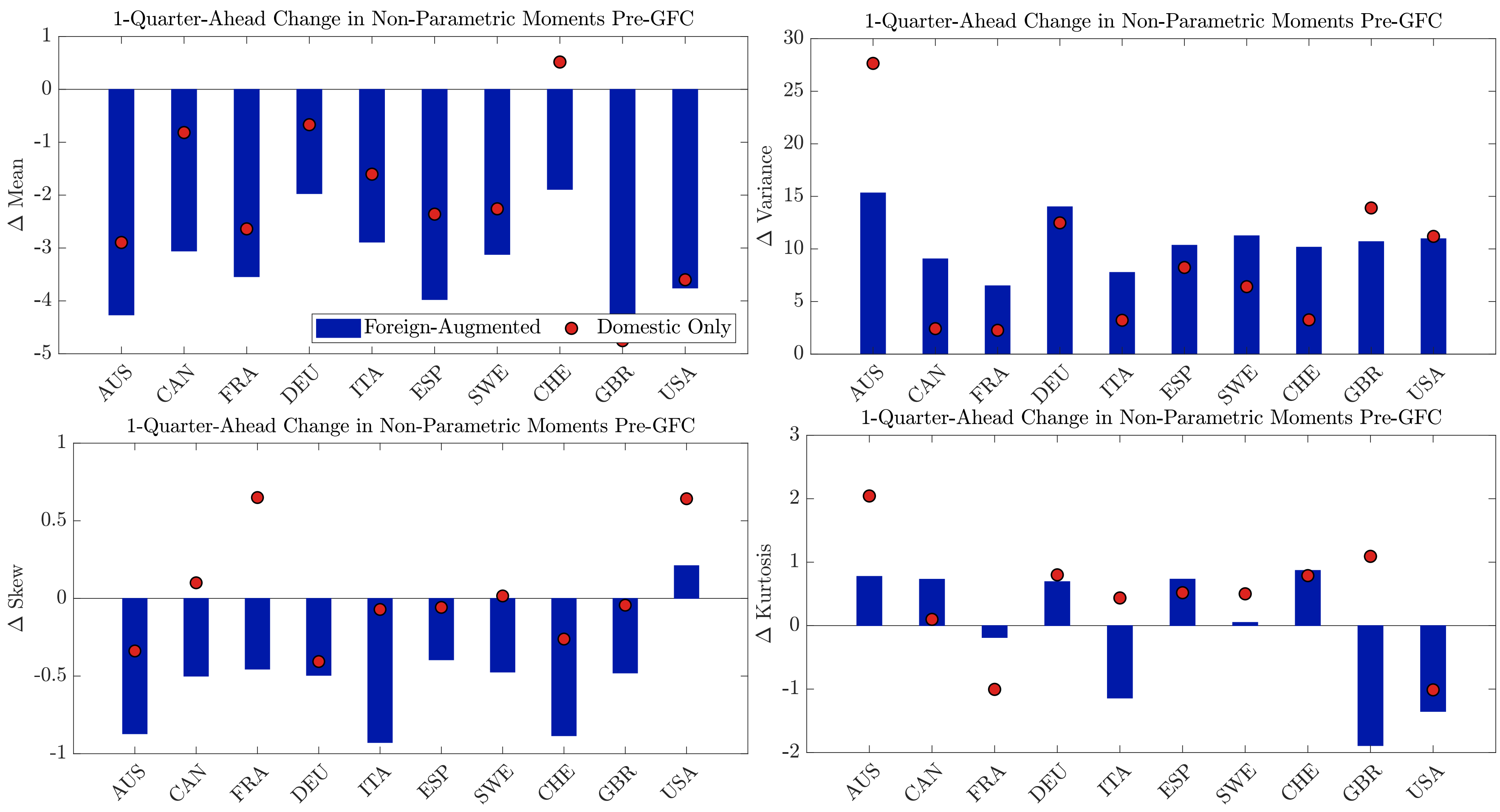
B.10 Out-of-Sample PITs

Figures 14 and 15 report the 1- and 4-quarter-ahead PITs for the 10 countries in turn, supporting the discussion in Section 3.3.2.

B.11 Time Variation in Out-of-Sample Moments

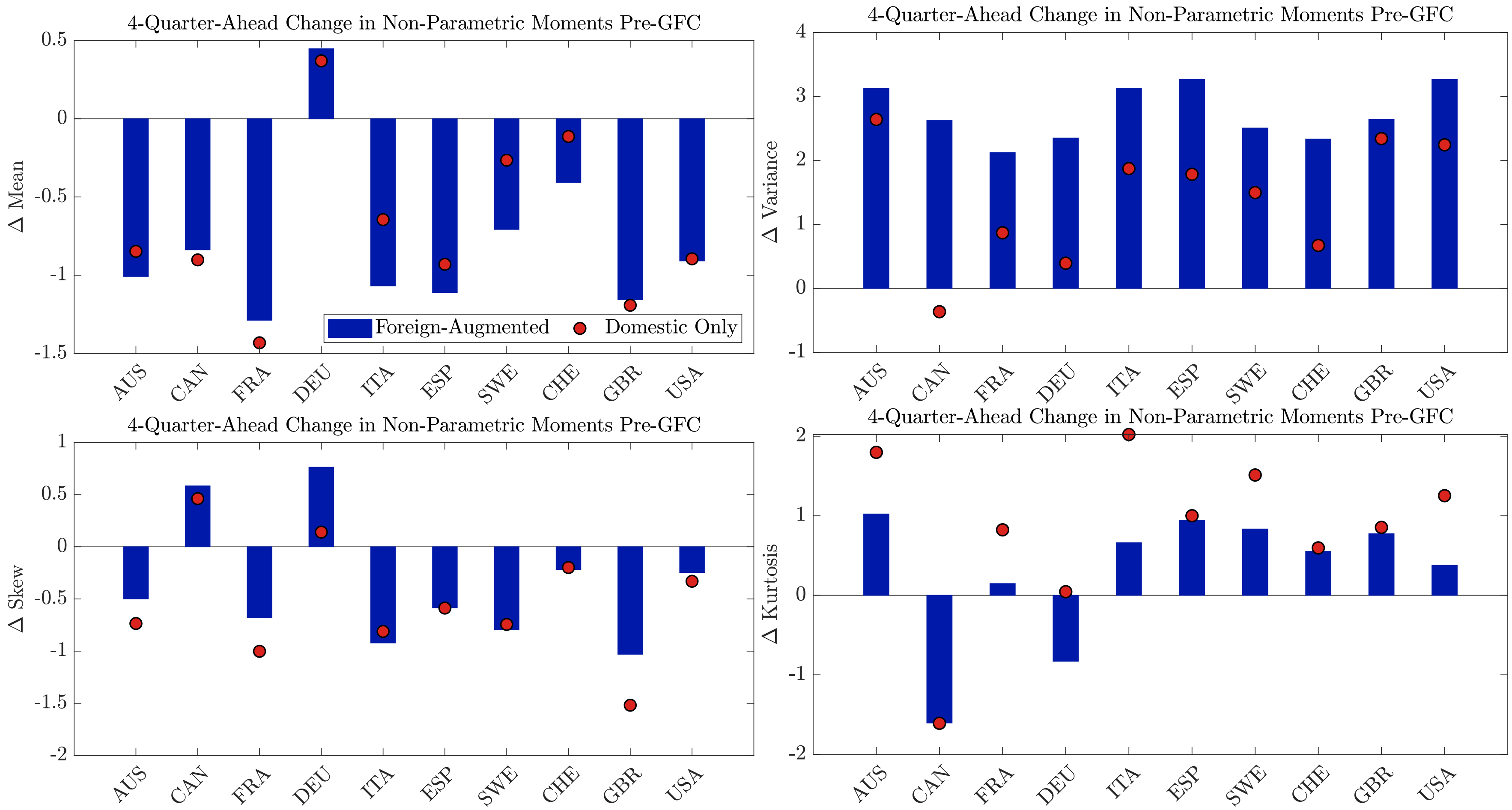
Figures 16 and 17 present estimated changes in moments across countries prior to the GFC, estimated out-of-sample.

Figure 10: Estimated change in 1-quarter-ahead GDP moments: 1997Q1 to 2006Q1



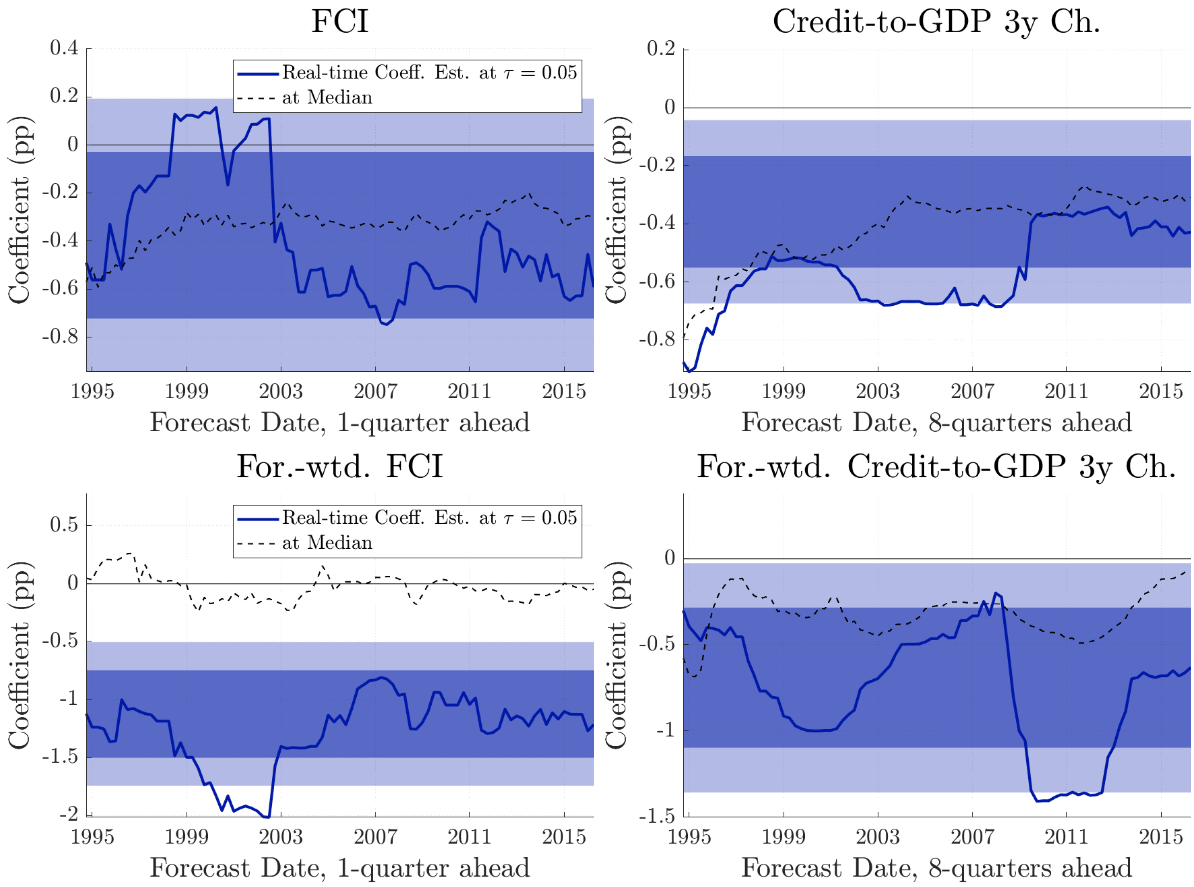
Note: Estimated change in the moments of 1-quarter-ahead annual average real GDP growth between 1997Q1 and 2006Q1 across countries. Blue bars represent estimates from model that includes foreign covariates. Red dots represent estimates from model that excludes foreign covariates.

Figure 11: Estimated change in 4-quarter-ahead GDP moments: 1997Q1 to 2006Q1



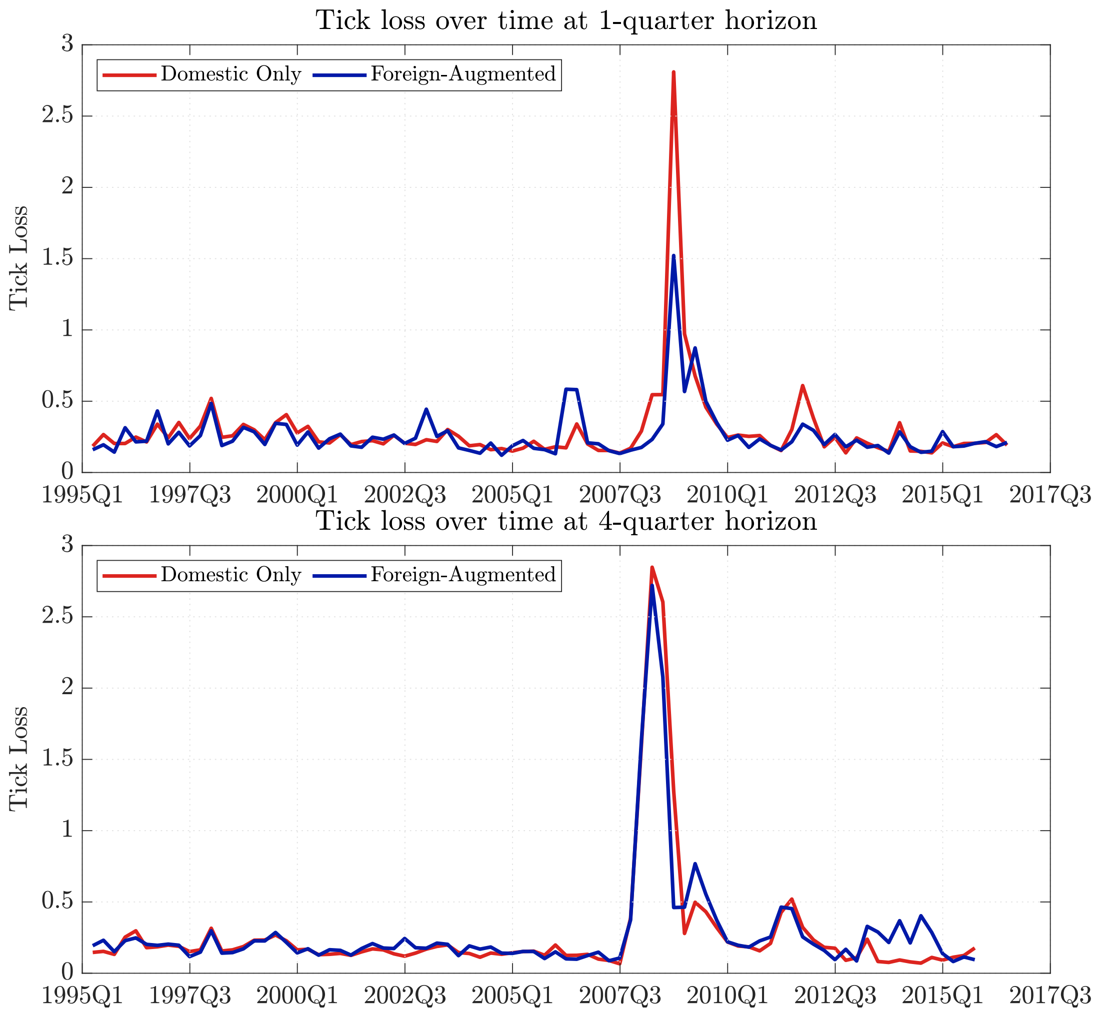
Note: Estimated change in the moments of 4-quarter-ahead annual average real GDP growth between 1997Q1 and 2006Q1 across countries. Blue bars represent estimates from model that includes foreign covariates. Red dots represent estimates from model that excludes foreign covariates.

Figure 12: Real-time estimates of the association between domestic and foreign-weighted indicators and the 5th (50th) percentile of GDP growth



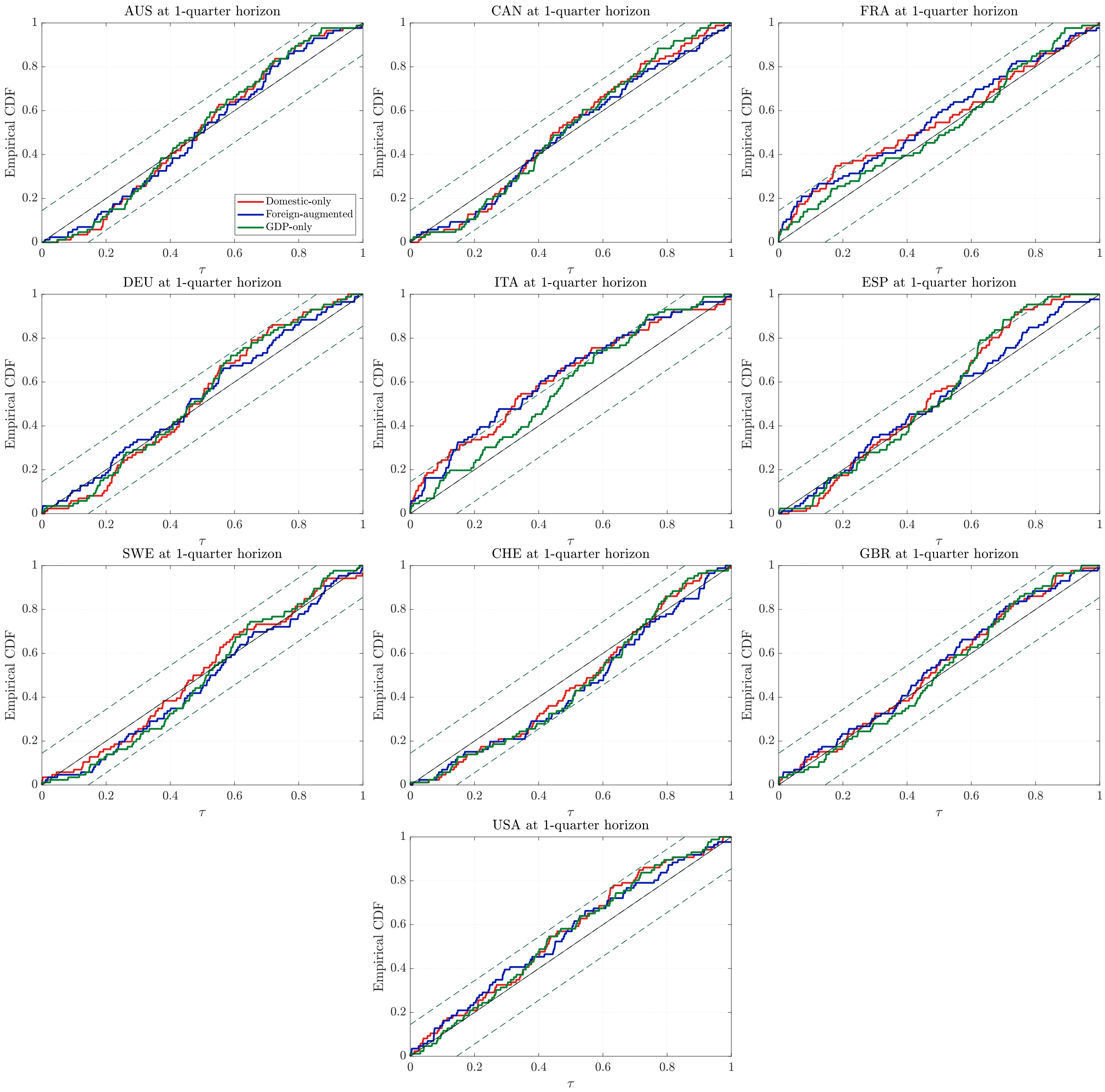
Note: Estimated real-time association between one standard deviation change in each indicator at each forecast date and 5th percentile (blue line) and median (black dashed line) of annual average real GDP growth at 1-quarter horizon for domestic and foreign-weighted financial market volatility and h = 8-quarter horizon for domestic and foreign-weighted credit-to-GDP. Light (dark) blue-shaded areas represent 95% (68%) confidence bands from block bootstrap procedure for the corresponding 5th percentile coefficient estimate over the full 1981Q1-2018Q4 sample.

Figure 13: Estimated out-of-sample tick loss at 5th percentile ($TL_t^h(\tau)$) over time



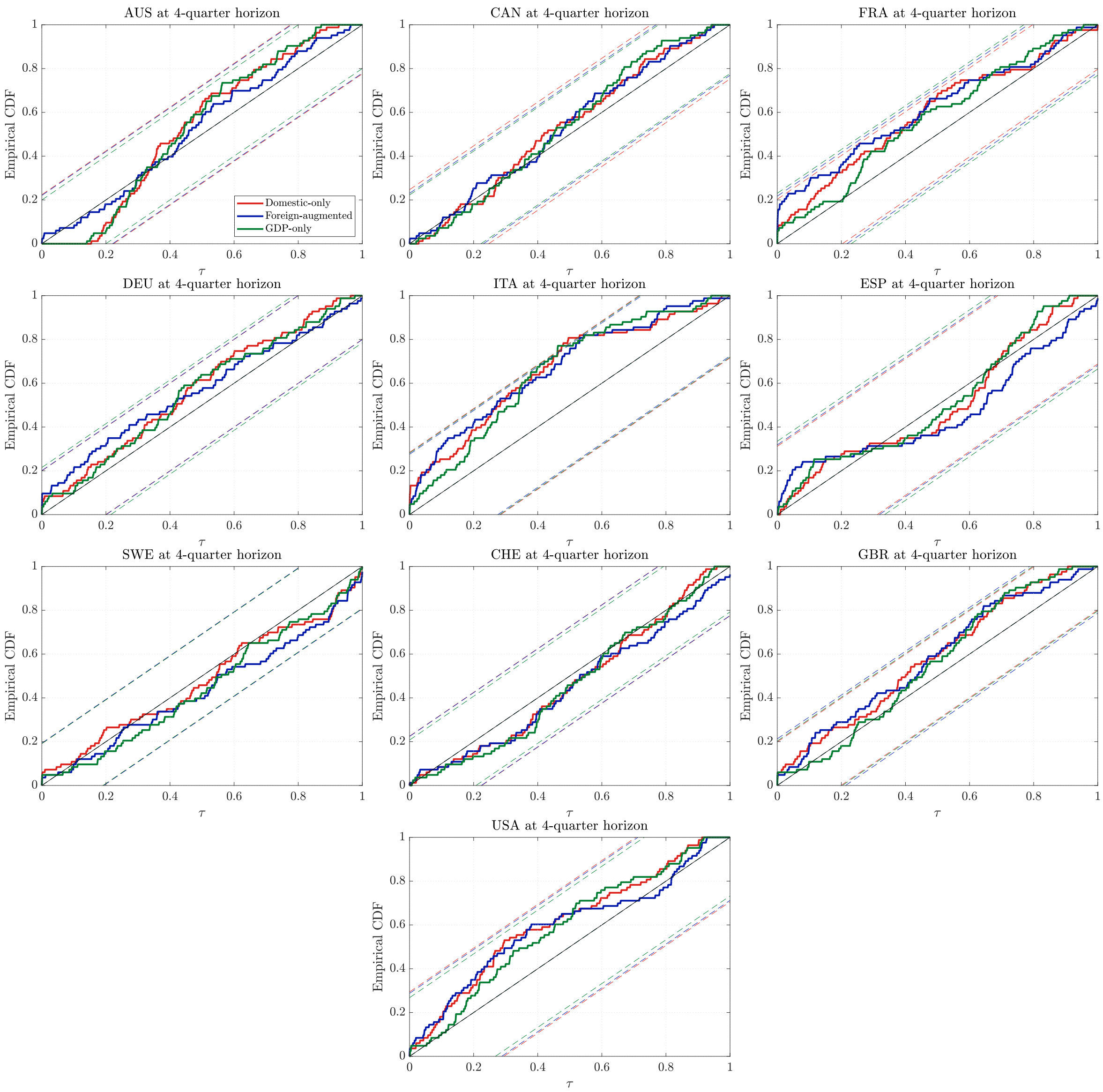
Note: Estimated average tick loss at the 5th percentile across countries over time and across horizons. The red line denotes estimates from the domestic-only specification and the blue line estimates from the foreign-augmented model.

Figure 14: Out-of-sample PITs: one-quarter ahead



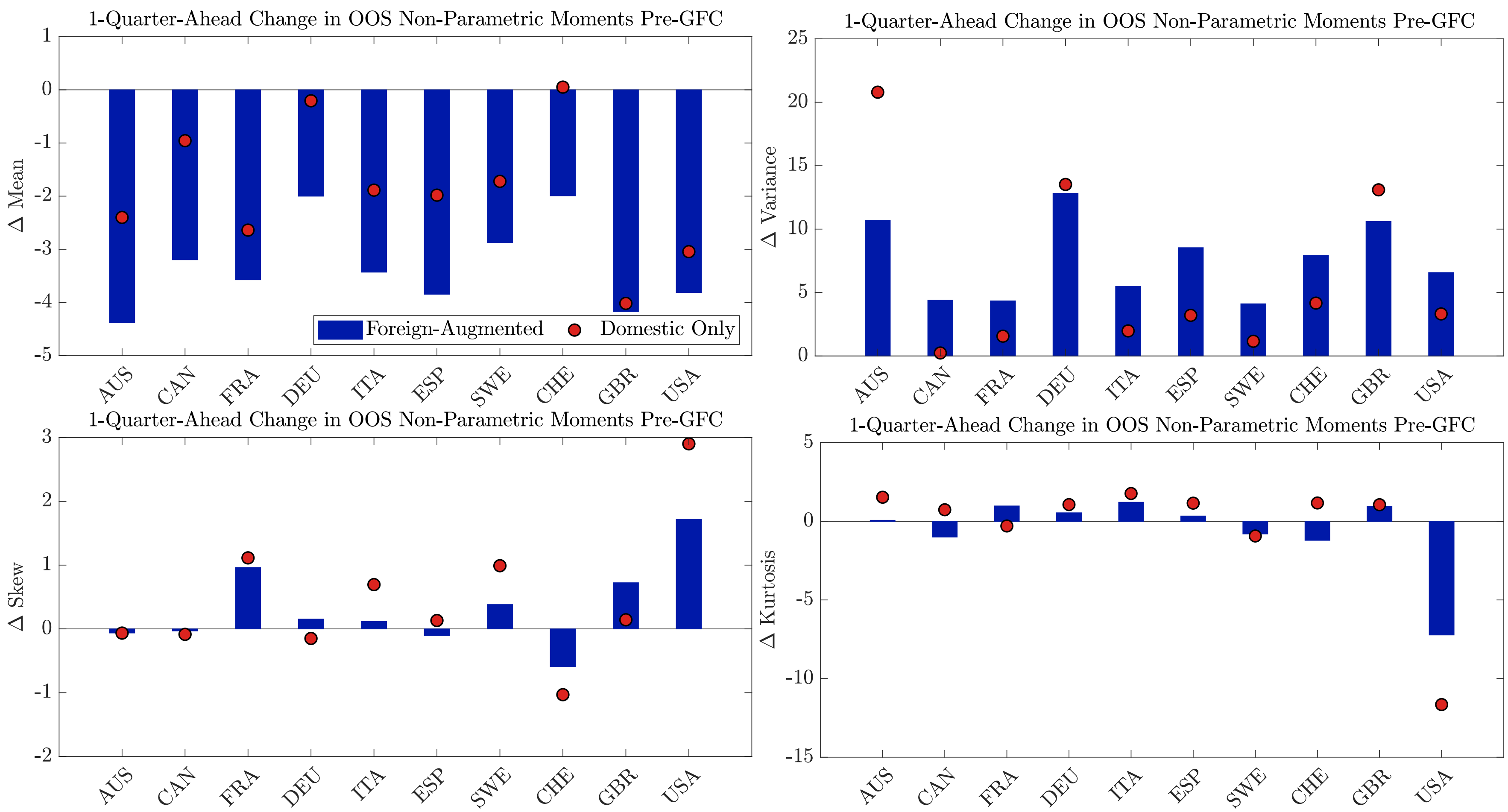
Note: Empirical cumulative distribution of the probability integral transform (PIT) of out-of-sample estimates at the 1-quarter-ahead horizon. Blue line shows the estimates from the foreign-augmented model, red line shows the estimates from the restricted domestic-only model, and green line shows estimates from GDP-only model. Dashed lines denote 95% confidence intervals, obtained using the method of Rossi and Sekhposyan (2019).

Figure 15: Out-of-sample PITs: one-year ahead



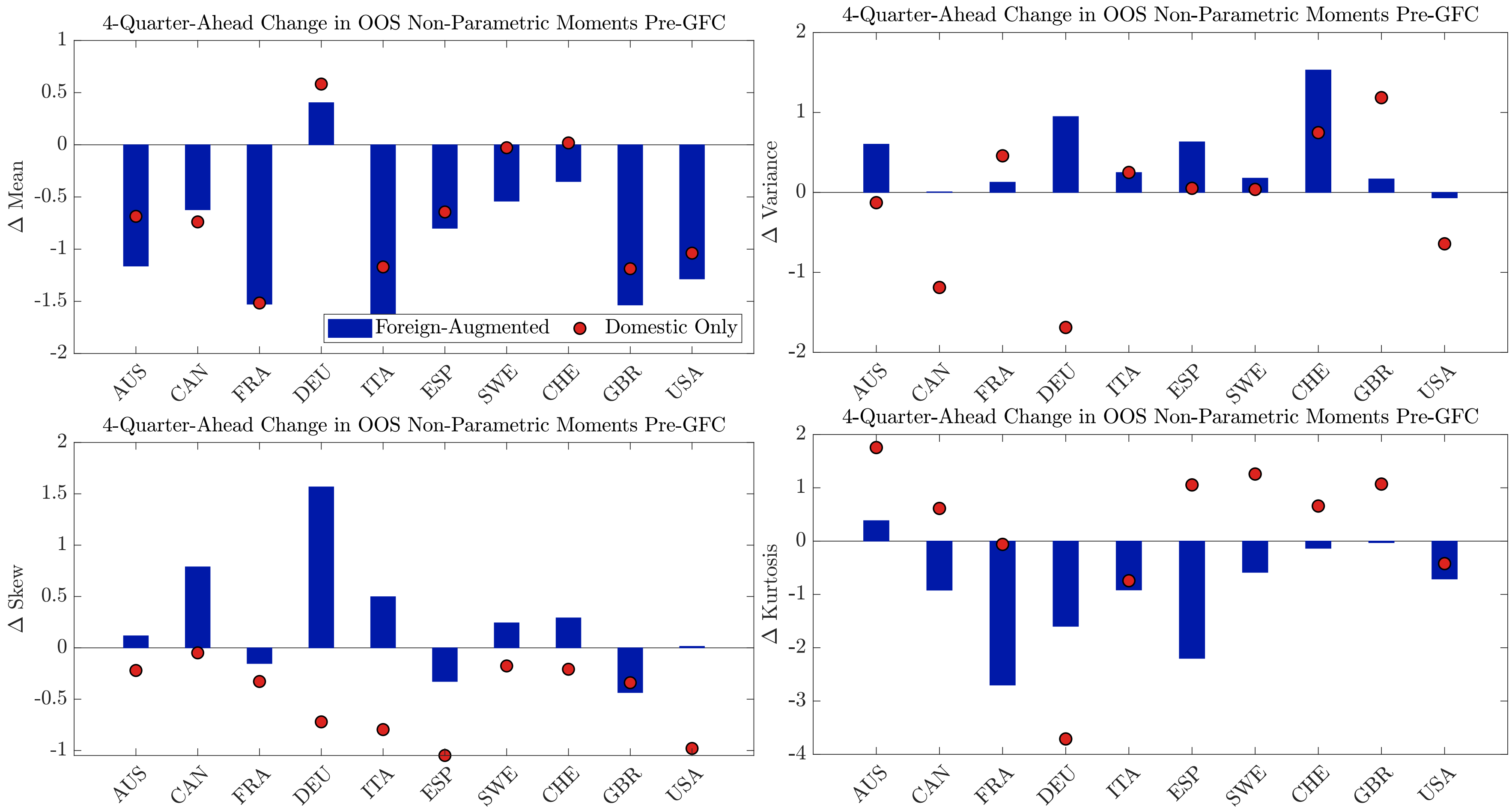
Note: Empirical cumulative distribution of the probability integral transform (PIT) of out-of-sample estimates at the 1-year-ahead horizon. Blue line shows the estimates from the foreign-augmented model, red line shows the estimates from the restricted domestic-only model, and green line shows estimates from GDP-only model. Dashed lines denote 95% confidence intervals, obtained using the method of Rossi and Sekhposyan (2019).

Figure 16: Estimated change in 1-quarter-ahead GDP moments: 1997Q1 to 2006Q1



Note: Estimated change in the moments of 1-quarter-ahead annual average real GDP growth between 1997Q1 and 2006Q1 across countries. Blue bars represent estimates from model that includes foreign covariates. Red dots represent estimates from model that excludes foreign covariates.

Figure 17: Estimated change in 4-quarter-ahead GDP moments: 1997Q1 to 2006Q1



Note: Estimated change in the moments of 4-quarter-ahead annual average real GDP growth between 1997Q1 and 2006Q1 across countries. Blue bars represent estimates from model that includes foreign covariates. Red dots represent estimates from model that excludes foreign covariates.

C Structural Contribution of Foreign Drivers

C.1 Structural Coefficient Estimates

In this sub-section, we report coefficient estimates at the 5th percentile for our alternate structural model described in Section 4. The coefficient estimates are similar to those in Figure 1, although the estimates of the impact of foreign credit-to-GDP and the foreign FCI on domestic GDP-at-Risk are now larger in magnitude. For example, in this specification, at peak, a one standard deviation increase in the foreign-weighted FCI is linked with a 1.5pp fall in the 5th percentile of GDP growth compared to a 1.1pp fall in Figure 1. This is because the first-stage orthogonalization allows us to capture the contemporaneous impact of foreign variables on domestic covariates—e.g., capturing the fact that a sharp tightening in global financial conditions can spill over contemporaneously to domestic financial conditions (and thereby worsen domestic GDP-at-Risk via this tightening in domestic conditions).

C.2 Two-Step Orthogonalization: Equivalence with Factor Model

As discussed in Section 4, the orthogonalization procedure applied to distinguish foreign shocks from domestic ones is commonplace in the empirical international macroeconomics literature (e.g., Dedola, Rivolta, and Stracca, 2017; Cesa-Bianchi and Sokol, 2022). It corresponds to a small-open economy assumption in which foreign events can contemporaneously influence domestic outcomes, but not vice versa.

In principle, an alternative proposal could be to decompose the k-th domestic predictor for country i, $x_{i,t}^{(k)}$, using a factor model of the form:

$$x_{i,t}^{(k)} = \lambda_i f_t^{(k)} + \eta_{i,t}^{(k)} \tag{10}$$

where $f_t^{(k)}$ represents a global factor and λ_i is the country-specific loading on it.

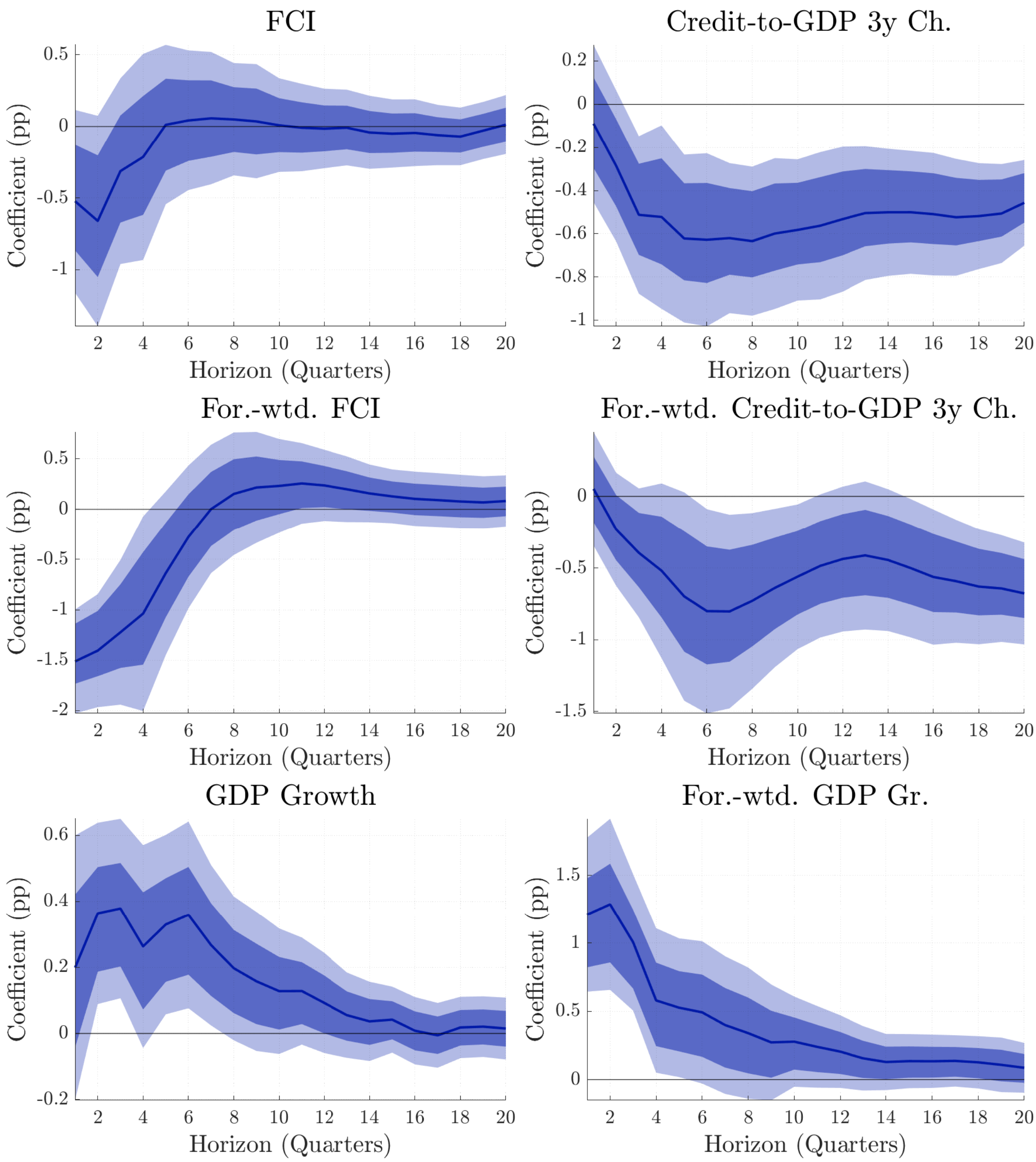
However, we can show that our foreign-weighted variable $x_{i,t}^{*(k)}$ is observationally equivalent to the factor $f_t^{(k)}$ up to a scalar, so demonstrating the equivalence between our two-step procedure and the factor model. This result is not novel to our work. Cesa-Bianchi et al. (2019b) show this in a GVAR setting. We show how this equivalence extends to the quantile regression setting.

The logic underpinning the observational equivalence is as follows. First, recall the definition of the k-th foreign-weighted predictor:

$$x_{i,t}^{*(k)} = \sum_{j=1}^{N} \omega_{i,j,t} x_{j,t}^{(k)} \tag{11}$$

and suppose that the k-th predictor for country i has the factor structure in equation (10).

Figure 18: Association between orthogonalized vulnerability indicators and GDP growth across horizons at the 5th percentile



Note: Solid blue lines denote estimated association between one standard deviation change in each indicator at time t with 5th percentile of average annual real GDP growth at each quarterly horizon. Light (dark) blue-shaded areas represent 90% (68%) confidence bands around these estimates from block bootstrap procedure.

Substituting the factor-model definition (10) into (11) yields:

$$x_{i,t}^{*(k)} = f_t^{(k)} \sum_{j=1}^{N} \omega_{i,j,t} \lambda_j^{(k)} + \overline{\eta}_{\omega,t}^{(k)}$$

where $\overline{\eta}_{\omega,t}^{(k)} \equiv \sum_{j=1}^{N} \omega_{i,j,t} \eta_{j,t}^{(k)}$.

Assuming that the loadings $\lambda_i^{(k)}$ are distributed independently across i and from the common shock $f_t^{(k)}$ for all i and t, with non-zero mean $\lambda^{(k)}$ and satisfy the following conditions:

$$\lambda^{(k)} = \sum_{j=1}^{N} \omega_{i,j,t} \lambda_j^{(k)} \neq 0 \quad \text{and} \quad \sum_{j=1}^{N} {\lambda_j^{(k)}}^2 = \mathcal{O}(N)$$

then the model can be written as:

$$x_{i,t}^{*(k)} = \lambda^{(k)} f_t^{(k)} + \overline{\eta}_{\omega,t}^{(k)}$$

Under the following two further assumptions:

- that the weights $\omega_{i,j,t}$ for $i = 1, ..., N$ are such that $\sum_{j=1}^{N} \omega_{i,j,t} = 1$ and satisfy *granularity* conditions, which requires individual countries' contributions to the foreign-weighted variable to be of order $1/N$, i.e., that $||\mathbf{w}_{i,t}|| = \mathcal{O}(N^{-1})$ and $\frac{\omega_{i,j,t}}{||\mathbf{w}_{i,t}||} = \mathcal{O}(N^{-1/2})$; and
- that country-specific shocks $\eta_{i,t}^{(k)}$ have zero means, finite variances and are serially uncorrelated, and denoting the the covariance matrix of the $N \times 1$ vector $\boldsymbol{\eta}_t^{(k)} = [\eta_{1,t}^{(k)}, ..., \eta_{N,t}^{(k)}]'$ by $\boldsymbol{\Sigma}_{\eta\eta} = \text{var}(\boldsymbol{\eta}_t^{(k)})$ with $\varrho_{\max}(\boldsymbol{\Sigma}_{\eta\eta}) = \mathcal{O}(1)$;

then it follows that $\text{var}(\overline{\eta}_{\omega,t}^{(k)}) = \mathcal{O}(\mathbf{w}_{i,t}'\mathbf{w}_{i,t}) = \mathcal{O}(N^{-1})$ and hence $\overline{\eta}_{\omega,t}^{(k)} = \mathcal{O}_p(N^{-1/2})$, allowing us to recover $f_t^{(k)}$ from $x_{i,t}^{*(k)}$ up to a scalar $\lambda^{(k)}$ using:

$$f_t^{(k)} = \frac{1}{\lambda^{(k)}} x_{i,t}^{*(k)} + \mathcal{O}(N^{-1/2})$$

thus proving the observational equivalence.

C.3 Towards a Structural Decomposition

In this sub-section, we report exemplar decompositions from our orthogonalization procedure.

Figure 19 shows the orthogonalized decomposition for the estimated 5th percentile of 3-year-ahead UK real GDP growth. The orthogonalized decomposition suggests that the estimated fall in UK 3-year GDP-at-Risk in the run-up to the 1990-1991 recession was predominantly driven by domestic drivers (red bars). Foreign drivers (blue bars) played a limited role. Following this recession, these factors reversed with the estimated rise in the 5th percentile of UK 3-year GDP growth supported by both domestic and foreign factors.

Tail risks built up substantially over the 2000s though, driven almost entirely by a build-up in foreign-weighted credit-to-GDP. This accords with the well established view that the GFC had global origins, driven by worldwide trends in an increasingly interconnected international financial system.

Since the GFC, these drivers of tail risks have again reversed, likely tempered by enhanced macroprudential policy toolkits and global monitoring of the financial system.

Figure 20 presents the comparable decomposition for German 3-year GDP-at-Risk. The relative evolution of domestic and foreign shocks in the run-up to the GFC is particularly notable for Germany. Domestic factors are associated with improvements in the left tail of the GDP growth distribution from 2004 to 2008. In contrast, foreign-weighted indicators are associated with a worsening in tail risk over the same period. In sum, these foreign factors dominate and contribute to an overall fall in fitted GDP-at-Risk over the period, exemplifying the importance of accounting for global influences when monitoring macro-financial risks.

Robustness We also present details of the additional robustness exercises we run to complement the structural decompositions in Section 4. Specifically, we estimate structural decompositions from two model variants, in addition to the baseline model.

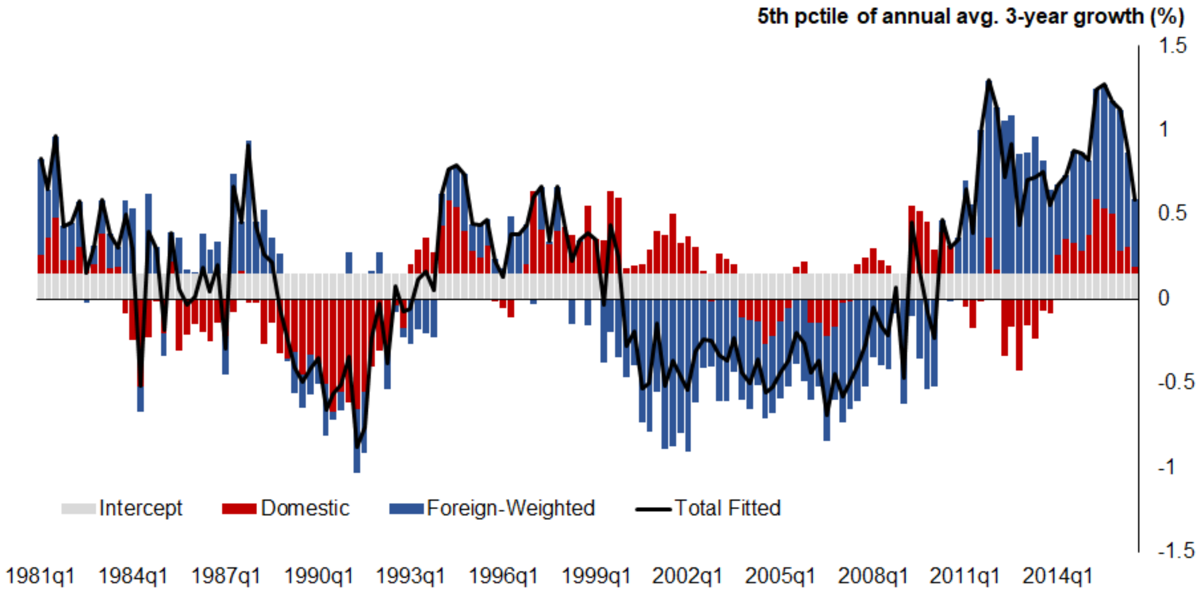
First, we re-estimate our baseline model, weighting foreign variables using bilateral financial linkages measured using BIS International Banking Statistics.

Second, we estimate an extended model, akin to that in Aikman et al. (2019). Here, the domestic variable set includes 3-year house price growth, the current account, bank capital ratios, 1-year CPI inflation and the 1-year change in central bank policy rates, in addition to our baseline domestic indicators (3-year change in credit-to-GDP growth and lagged quarterly real GDP growth).

The estimated share of variation in fitted values attributable to foreign shocks $ForShare_i^h(\tau)$, defined in equation (9), at $h = 1, 4, 12$ and $\tau = 0.05, 0.5$ from these two models, alongside the baseline, are presented in Table 12.

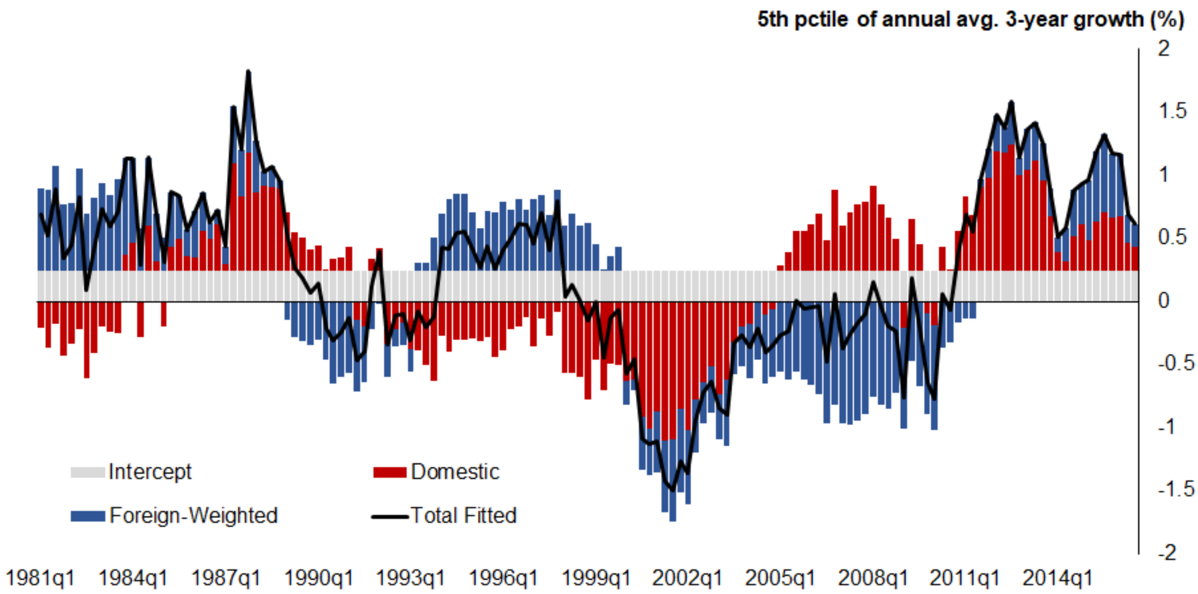
In all three models, a substantial share of variation in estimated percentiles of GDP growth is attributable to foreign shocks. Moreover, foreign factors generally exert a larger influence on fitted values at the left tail of the GDP distribution, i.e., the 5th percentile, than at the median, corroborating the results in Table 6. Although the foreign share is lowest for the extended model, this is unsurprising given that it includes more domestic covariates than the baseline or its financially-weighted variant. Even so, the results in Table 12 indicate that, across models, between 39 and 56% of variation in the 5th percentile of 3-year GDP growth is attributable to foreign shocks.

Figure 19: Estimated orthogonalized decomposition of UK GDP-at-Risk at the 3-year horizon



Note: Solid black line denotes estimated 5th percentile of annual average 3-year-ahead GDP growth at each point in time. The bars show the contribution of domestic and foreign-weighted indicators to that total from estimates of equation (8).

Figure 20: Estimated orthogonalized decomposition of German GDP-at-Risk at the 3-year horizon



Note: Solid black line denotes estimated 5th percentile of annual average 3-year-ahead GDP growth at each point in time. The bars show the contribution of domestic and foreign-weighted indicators to that total from estimates of equation (8).

Table 12: Share of Variation in Fitted Values (%) Attributes to Foreign Shocks Across Horizons

	(1) Baseline						(2) Financial Weights						(3) Extended Specification					
	$h=1$		$h=4$		$h=12$		$h=1$		$h=4$		$h=12$		$h=1$		$h=4$		$h=12$	
Country	$\tau=0.05$	0.5	0.05	0.5	0.05	0.5	0.05	0.5	0.05	0.5	0.05	0.5	0.05	0.5	0.05	0.5	0.05	0.5
AUS	97.73	86.42	91.73	80.29	57.86	61.38	94.76	88.09	92.36	80.06	46.36	36.62	88.67	79.69	77.84	54.60	38.09	44.81
CAN	98.39	87.60	89.03	79.26	42.69	46.86	95.72	90.03	92.36	80.38	39.01	29.54	90.61	83.21	80.91	57.30	33.43	34.13
FRA	97.39	91.20	90.47	83.27	43.77	46.84	90.73	91.19	93.02	82.59	41.52	31.60	85.84	87.43	79.21	59.83	36.59	35.73
DEU	97.19	86.63	89.76	79.78	51.66	53.34	90.44	87.49	92.85	80.53	43.82	32.54	95.55	84.91	81.31	62.73	40.59	43.61
ITA	97.43	91.76	95.35	89.16	63.01	68.38	84.92	90.80	94.35	86.09	48.90	33.12	88.98	87.20	85.55	63.06	47.17	51.63
ESP	97.74	87.30	91.86	80.88	54.34	59.92	91.96	87.63	92.98	80.70	51.72	42.35	89.34	81.77	79.74	60.85	37.18	48.99
SWE	97.03	87.49	94.37	85.17	65.71	67.68	86.09	86.14	92.57	81.08	44.96	33.93	83.44	79.35	80.19	58.07	46.95	50.26
CHE	98.71	91.60	91.61	85.60	45.01	45.93	94.58	90.43	92.43	82.50	36.87	26.39	88.22	83.99	72.89	49.18	27.24	26.52
GBR	98.68	89.28	95.67	87.82	76.65	80.30	95.67	89.17	95.76	86.41	59.20	50.03	91.34	83.53	81.27	60.02	43.42	49.12
Avg.	*97.81*	*88.81*	*92.21*	*83.47*	*55.63*	*58.96*	*91.65*	*89.00*	*93.19*	*82.26*	*45.82*	*35.12*	*89.11*	*83.45*	*79.88*	*58.40*	*38.96*	*42.75*

Share of variation at the 5th percentile ($\tau = 0.05$) and median ($\tau = 0.5$) of country-GDP distributions at different horizons: $h = 1$ (1 quarter), $h = 4$ (1 year), and $h = 12$ (3 years). Share definition in equation (9). Shares constructed from three models in which domestic indicators are orthogonalised with respect to all foreign indicators, akin to a small-open economy assumption for domestic countries.