Exchange Rates, Yield Curves and Transitory Risk*

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July 29, 2024

Abstract

We document that currencies with a steeper yield curve tend to depreciate against the US dollar, independently of interest-rate differentials, especially at 2 to 4-year horizons. Using survey data, we demonstrate that this relationship is driven by expectations of macroeconomic fundamentals reflected in the yield-curve slope. Within a no-arbitrage, preference-free framework, we highlight the role of unspanned transitory risk factors as drivers of both cross-country differences in yield-curve slopes and exchange-rate risk premia. These 'hidden' factors can emerge endogenously in models of domestically incomplete markets, explaining both our documented relationship and the 'disconnect' of exchange rates from interest rates.

JEL Codes: E43, F31, G12.

Key Words: Bond Yields; Exchange-Rate Risk Premia; Incomplete Markets; Term Structure; Uncovered Interest Parity.

*We are especially grateful to Giancarlo Corsetti for many helpful discussions. We also thank anonymous referees, Gianluca Benigno, Ambrogio Cesa-Bianchi, Joe Chen, Mikhail Chernov, Luca Dedola, Pierre De Leo (discussant), Pierre-Olivier Gourinchas, Matteo Maggiori, Thomas Nitschka (discussant), Ilaria Piatti (discussant), Katrin Rabitsch, Lucio Sarno, Sanjay Singh, Alan Taylor, and Adrien Verdelhan, as well as presentation attendees at the Bank of Canada, Bank of England, Centre for Central Banking Studies, Banque de France-Bank of England International Macroeconomics Workshop, CRETE 2019, European Economic Association Annual Conference 2019, Federal Reserve Bank of New York, Money, Macro and Finance Annual Conference 2019, the Royal Economic Society Annual Conference 2019, University of Nottingham, University of Cambridge, the 10th Banca d'Italia-European Central Bank Workshop on Exchange Rates, the 37th International Conference of the French Finance Association, UC Davis, the American Economic Association Annual Meetings 2023, Texas State University, the 5th Liquidity in Macroeconomics Workshop (Federal Reserve Bank of San Francisco), the Bank of Canada-Federal Reserve Bank of San Francisco Conference on Fixed Income Markets 2023, and the International Association for Applied Econometrics Annual Conference 2023 for useful comments. The paper was previously presented with the title "Exchange-Rate Risk and Business Cycles". Marin acknowledges the Keynes Fund at the University of Cambridge (Project JHUK) for financial support on this project. The views expressed in this paper are those of the authors, and not necessarily those of the Bank of England.

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1 Introduction

A long-standing literature in international macroeconomics has questioned whether exchange rates can be connected to macroeconomic fundamentals, highlighting an 'exchange-rate disconnect' (Meese and Rogoff, 1983; Itskhoki and Mukhin, 2021).[1] In parallel, leading contributions in asset pricing have taken exchange-rate puzzles at face value, evaluating the restrictions they impose on the pricing of risk (e.g., Backus, Foresi, and Telmer, 2001; Lustig, Stathopoulos, and Verdelhan, 2019). The most prominent of these is the failure of the uncovered interest parity (UIP) condition, which predicts that high interest-rate currencies should depreciate to equalize exchange-rate-adjusted returns (Fama, 1984) and, relatedly, the lack of predictability of exchange rates by short-term interest-rate differentials (e.g., Chernov and Creal, 2023). To the extent that macroeconomic fundamentals relevant for exchange-rate fluctuations are captured in interest-rate differentials, the failure of UIP is frequently viewed as corroborating evidence of a disconnect.

In this paper, we shift focus from spot-yield differentials to the 'shape' of the yield curve, particularly its slope. We show that fluctuations in relative yield-curve slope across countries explain exchange-rate dynamics. Drawing on a literature identifying the yield-curve slope as a leading indicator of macroeconomic outcomes (Estrella and Hardouvelis, 1991; Estrella and Mishkin, 1998; Estrella, 2005), we investigate whether cross-country slope differentials offer scope to 'reconnect' currency moves to fundamentals both theoretically, within a no-arbitrage framework, and empirically, using survey data on macroeconomic expectations.

Our starting point is the canonical UIP regression, which we augment with cross-country differences in yield-curve factors. We first document that advanced economies with a relatively steep yield curve tend to depreciate in excess of UIP, *vis-à-vis* the US, with the relationship strongest at 2 to 4-year horizons. Then, we isolate the specific contribution of the relative yield-curve slope in explaining variation in bond risk premia and exchange-rate risk premia (ERRP), across holding periods and bond maturities, extending the empirical analysis in Lustig et al. (2019). Our results suggest the relationship between exchange rates and the relative yield-curve slope is predominantly driven by the ERRP and is orthogonal to interest-rate differentials. In both the augmented UIP regression and the decomposition of risk premia, we identify a tent-shaped relationship, across holding periods, for a range of bond maturities. This result is robust to a range of specification changes, including to the inclusion of liquidity yields (Du, Im, and Schreger, 2018; Engel and Wu, 2022) (i.e., the non-monetary return that government bonds provide because of their safety, ease of resale, and value as collateral).

To interpret our results through the lens of theory, we then ask two questions: (i) what risks drive the relationship between yield curves and ERRP, and (ii) can this be consistent with the disconnect of exchange rates from short-term interest rates?

Our answer to the first question is that, within a no-arbitrage, preference-free framework,

[1] Naturally, this has also motivated a literature which attempts to 'reconnect' currency moves to fundamentals, using production data (Colacito, Riddiough, and Sarno, 2020), capital flows (Lilley, Maggiori, Neiman, and Schreger, 2022), or productivity-news shocks (Chahrour, Cormun, De Leo, Guerron-Quintana, and Valchev, 2021).

transitory innovations (Alvarez and Jermann, 2005) are crucial. They generate a negative autocorrelation of stochastic discount factors (SDFs) and ensure that yield curves slope upwards on average (Piazzesi and Schneider, 2007). Facing shocks today, investors *expect* booms—periods in which consumption has low marginal value—to be followed by busts—periods in which consumption has high marginal value—in line with a notion of 'business-cycle risk'.[2] Extending this logic to a two-country setting, we show this implies that cross-country differences in yield-curve slopes will reflect asymmetries in business-cycle risk. In turn, this correlates with ERRP, which reflect cross-country differences in the volatility of SDFs—the permanent component of which has been shown to be close to zero (Lustig et al., 2019), thus leaving transitory risk as the primary driver.

We test this prediction empirically using survey expectations from *Consensus Economics* for our set of advanced countries and document that expectations of GDP and inflation, relative to the US, explain a significant portion of variation in the *relative* yield-curve slope. We then estimate the relationship between the (fitted) relative yield-curve slope, explained by variation in macroeconomic expectations, and ERRP—comparing it to the unexplained component. Strikingly, we find that only movements in the fitted component, which captures cross-country asymmetries in business-cycle dynamics, are a significant predictor of ERRP at intermediate horizons. The unexplained component is only significant at short horizons.

To address the second question, we show that our regression evidence specifically suggests the presence of transitory innovations which have offsetting effects on the mean and variance of SDFs, but matter for SDF autocorrelation. Building on this, we consider a simple model which rationalizes *both* the relationship between relative yield-curve slope and ERRP in closed form *and* the lack thereof between exchange rates and spot-yield differentials. The model extends the SDF process considered in Alvarez and Jermann (2005) and Backus, Chernov, and Zin (2014), which resembles the canonical Vasicek (1977) model for the term-structure of interest rates, and we use it to undertake a risk-accounting exercise. We allow for three sources of risk: (i) an uncorrelated permanent shock, (ii) a transitory shock and (iii) a transitory shock which is 'hidden' from the short rate. Both risk-factors (i) and (iii) are not fully reflected in yields and so maintain a disconnect between ERRP and spot-yield differentials. But only risk-factor (iii) can drive the relationship between the relative yield-curve slope and ERRP, confirming that their association can be consistent with an apparent disconnect.

Furthermore, we illustrate that these hidden factors arise as *equilibrium* outcomes in a class of incomplete-markets models. Consider a framework where there are two investors in the domestic economy. The marginal investor trades in both Home and Foreign bonds and is exposed to two transitory factors, while the other only invests in Home bonds and is exposed only to the first factor. In this case, the absence of arbitrage *requires* that the second factor be hidden from the bond price (i.e., it be unspanned), since investors must agree on pricing of the domestic bond. Indeed, this framework relates to models with preferred habitats (e.g., Gourinchas, Ray, and Vayanos, 2022; Greenwood, Hanson, Stein, and Sunderam, 2023), but our approach allows

[2]This is consistent with Basu et al. (2021) who identify a 'risk shock' that drives a large portion of aggregate comovement over the business-cycle and contributes to a positive yield-curve slope.

us to specifically focus on spanning properties and the horizon of risk. Alternatively, we explain that our framework may capture heterogeneous expectations of agents over fundamentals and expectation formation need not be rational.[3]

We conclude by showing that our findings survive in a class of dynamic asset-pricing models that allow for time-varying volatility (e.g., Cox, Ingersoll, and Ross, 1985) and shares common implications with prominent equilibrium models in the literature—i.e., habits (Campbell and Cochrane, 1999) and long-run risk (Bansal and Shaliastovich, 2013). For this, we build on a model of central tendency (Balduzzi, Das, and Foresi, 1998; Ang and Chen, 2010) to consider a factor which drives the bond premium (through the autocorrelation of SDFs), but leaves interest rates unchanged.

Related Literature. Our work builds on the classic literature on the forward-premium puzzle (Hansen and Hodrick, 1980; Fama, 1984), and analyses of UIP across time (Engel, 2016) and horizons (Chinn and Meredith, 2005; Chinn and Quayyum, 2012; Chernov and Creal, 2020). We focus on the cross-time component of UIP failures, which Hassan and Mano (2019) show is an important component of currency predictability. Specifically, our empirical setup builds on Lustig et al. (2019). While they show that, for a given one-month holding period, the term structure of carry trade is decreasing, we extend their specification across holding periods to show that the relative slope is a significant predictor of ERRP at business-cycle horizons, for a range of maturities, orthogonally to interest-rate differentials. Several other papers (e.g., Engel, 2016; Valchev, 2020) focus on the horizon variation in the path for exchange rates, conditional on interest rates, but we extend their analysis to the term structure.

A number of papers show that yield-curve factors can significantly predict ERRP, but many focus on horizons shorter than ours (less than 2 years) (Ang and Chen, 2010; Gräb and Kostka, 2018). While Chen and Tsang (2013) also study longer horizons, they only find significance at short ones. We attribute this difference to the fact Chen and Tsang (2013) capture relative yield-curve factors by directly estimating Nelson and Siegel (1987) decompositions from relative interest-rate differentials, thus assuming common factor structures across countries. In contrast, we construct proxies for factors using yield curves estimated on a country-by-country basis, allowing factor structures to be country-specific.

We argue that yield curves reflect transitory business-cycle risks, and show that this can explain time-series variation in ERRP. Colacito, Riddiough, and Sarno (2020) also attribute a role to business cycles in explaining ERRP, but in the cross-section—sorting currencies by output gap. Insofar as a high output gap contributes to a steeper yield-curve slope, our findings are consistent. However, whilst the output gap is backward-looking, our paper assesses the ability of the forward-looking term structure to explain ERRP. We further use measures of macroeconomic expectations, contributing to a growing literature using forecasts from *Consen-*

[3]Tying to our empirics, this is consistent with a literature highlighting that survey forecast errors are autocorrelated (Bordalo, Gennaioli, Ma, and Shleifer, 2020; Candian and De Leo, 2023). This may explain why long-term bond prices, which correlate with surveys, are better able to reconnect exchange rates with, at least, subjective expectations of macro variables.

sus Economics to assess exchange-rate anomalies (e.g., Candian and De Leo, 2023; Stavrakeva and Tang, 2023), extending the analysis of Bansal and Shaliastovich (2013) who find that the *country-specific* survey expectations explain variation in *country-specific* yield-curve slopes.

Theoretically, we contribute to a literature using term structure models to explain ERRP (e.g., Lustig et al., 2019; Chernov and Creal, 2023). First, relative to these papers, we define transitory hidden factors and show they are required to rationalize the relationship between yield curves and ERRP. Second, while term-structure models are almost exclusively written under complete markets with a unique SDF, we contribute to a nascent literature on hidden factors Joslin, Priebsch, and Singleton (2014) and a small literature discussing market incompleteness (e.g., Bakshi, Crosby, Gao, and Hansen, 2023), illustrating how these factors arise as equilibrium outcomes in models of inomplete markets. Finally, turning to models of stochastic volatility, we build on Balduzzi et al. (1998) and Ang and Chen (2010) who construct an example where ERRP are forecastable only by factors present in the term structure but hidden from the short rate. We further highlight the difficulties the model must overcome to additionally the failure of UIP, positive yield curves and a relationship between bond premia and ERRP.

Outline. Section 2 presents the UIP regression augmented with yield-curve factors, before we use a generalized regression allowing for variable holding periods in Section 3. In Section 4, we lay out a preference-free, no-arbitrage framework to establish a link between yield curves and transitory risk, before testing it empirically using survey data. Section 5 rationalizes our findings, alongside the exchange-rate disconnect, with a term-structure model. Section 6 concludes.

2 Exchange Rates and the Yield-Curve Slope

To motivate our analysis, we estimate canonical UIP regressions across horizons augmented with relative yield-curve factors, presenting a novel empirical finding through the lens of a well-established framework. In doing so, we also describe the data used throughout.

2.1 Canonical UIP Regression

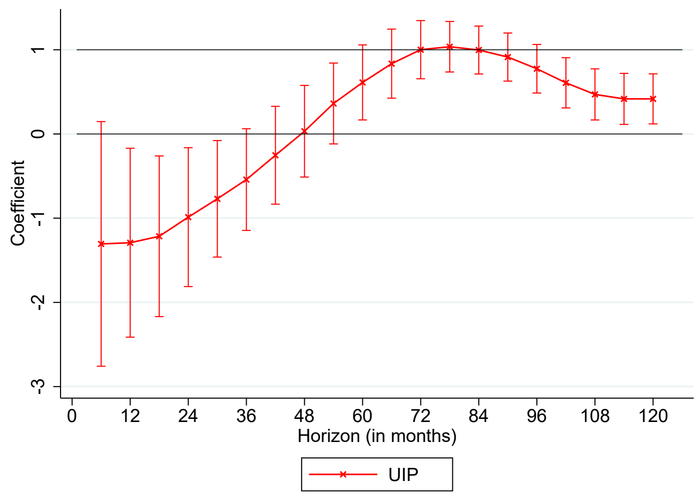
The UIP regression for κ-month-ahead exchange-rate changes is written as:

$$e_{j,t+\kappa} - e_{j,t} = \beta_{1,\kappa}\left(r^*_{j,t,\kappa} - r_{t,\kappa}\right) + f_{j,\kappa} + u_{j,t+\kappa} \tag{1}$$

where $e_{j,t} \equiv \log(\mathcal{E}_{j,t})$ is the (log) exchange rate of the Foreign country j *vis-à-vis* Home (base) currency at time t. It is defined as the Foreign price of a unit of base currency such that an increase in $e_{j,t}$ corresponds to a Foreign depreciation. $r^*_{j,t,\kappa}$ is the net κ-period continuously-compounded return in the Foreign country and $r_{t,\kappa}$ is the corresponding Home return. $f_{j,\kappa}$ is a country fixed effect and $u_{j,t+\kappa}$ is the disturbance.

Under the joint assumption of risk neutrality and rational expectations, the null hypothesis of UIP is that $\beta_{1,\kappa} = 1$ for all $\kappa > 0$ (and $f_{j,\kappa} = 0$ for all j and $\kappa > 0$). Empirical rejections of UIP

Figure 1: Estimated coefficients from canonical UIP regression at different horizons



Notes: Red crosses denote $\hat{\beta}_{1,\kappa}$ estimates from regression (1). Horizontal axis denotes the horizon κ in months. Regressions estimated using pooled monthly data for 6 currencies (AUD, CAD, CHF, EUR, JPY, GBP) against the USD from 1980:01 to 2019:12, including country fixed effects. 95% confidence intervals, calculated using Driscoll and Kraay (1998) standard errors, denoted by red bars around point estimates.

at short to medium horizons—i.e., finding $\hat{\beta}_{1,\kappa} \neq 1$ for small to medium κ—have regularly been used to motivate claims that interest rates do not adequately explain exchange-rate dynamics.

Data. We estimate regression (1) using exchange- and interest-rate data for 7 jurisdictions with liquid bond markets: Australia, Canada, Switzerland, Euro Area, Japan, UK, and US. The US is the base country among our sample of G7 currencies. To capture the term structure of interest rates in each region, we use nominal zero-coupon government bond yields of 6, 12, 18, ..., 120-month maturities. Yield curves are obtained from a combination of sources, including central banks and Wright (2011) (Appendix A), so our bond-yield panel is unbalanced. Nominal exchange rate data is from *Datastream*. We use end-of-month data from 1980:01 to 2019:12.

Results. Figure 1 plots UIP coefficient estimates $\hat{\beta}_{1,\kappa}$, which are also tabulated in Appendix B. Confidence bands around point estimates are derived from Driscoll and Kraay (1998) standard errors, which correct for heteroskedasticity and serial correlation. The coefficient estimates reinforce the view that the UIP hypothesis can be rejected at short to medium horizons, but is harder to reject at longer horizons. Point estimates are negative at 6 to 42-month tenors, indicating that high short-term interest rate currencies tend to appreciate, instead of depreciate. Longer-horizon point estimates are positive and close to unity, corroborating with, e.g., Chinn and Meredith (2005) and Chinn and Quayyum (2012).

2.2 Yield Curve-Augmented Regression

To illustrate the link between exchange rates and the yield-curve slope, we augment regression (1) with measures of the relative yield-curve slope $S^*_{j,t} - S_t$ and curvature $C^*_{j,t} - C_t$, estimating:

$$e_{j,t+\kappa} - e_{j,t} = \beta_{1,\kappa}\left(r^*_{j,t,\kappa} - r_{t,\kappa}\right) + \beta_{2,\kappa}\left(S^*_{j,t} - S_t\right) + \beta_{3,\kappa}\left(C^*_{j,t} - C_t\right) + f_{j,\kappa} + u_{j,t+\kappa} \tag{2}$$

for all κ, where $S^*_{j,t}$ ($C^*_{j,t}$) is the slope (curvature) of the Foreign-country-j yield curve at time t, and S_t (C_t) is the slope (curvature) of the base-country yield curve. Within this specification, the coefficient $\beta_{2,\kappa}$ captures the relationship between the relative slope and exchange rates that is orthogonal to interest-rate differentials and the relative curvature.

Along with the yield-curve level, the slope and curvature are known to capture a high degree of variation in bond yields (Litterman and Scheinkman, 1991). We proxy the relative level in regression (2) with the κ-period interest-rate differential $(r^*_{j,t,\kappa} - r_{t,\kappa})$. This ensures that the specification nests UIP, such that $\beta_{2,\kappa}$ captures the yield-curve slope's contribution *over and above* spot-yield differentials. Defining the *ex post* κ-period ERRP for Foreign currency as $rx^{FX}_{j,t,\kappa} \equiv r^*_{j,t,\kappa} - r_{t,\kappa} - (e_{j,t+\kappa} - e_{j,t})$ and combining with equation (2) yields:

$$rx^{FX}_{j,t,\kappa} = (1 - \beta_{1,\kappa})\left(r^*_{j,t,\kappa} - r_{t,\kappa}\right) - \beta_{2,\kappa}\left(S^*_{j,t} - S_t\right) - \beta_{3,\kappa}\left(C^*_{j,t} - C_t\right) - f_{j,\kappa} - u_{j,t+\kappa} \tag{3}$$

From this, we see that $\beta_{2,\kappa}$ can be interpreted as either the average Foreign depreciation (in percent) or the average *decrease* in the ERRP (in pp) associated with a 1pp increase in the slope of the Foreign yield curve relative to the US.

We measure the yield-curve slope and curvature in each region with proxies. We define the slope as the difference between the 10-year and 6-month yields, $S^*_{j,t} \equiv y^*_{j,t,10y} - y^*_{j,t,6m}$. We proxy the curvature using a butterfly spread, a function of 6-month, 5 and 10-year yields (Diebold and Rudebusch, 2013): $C^*_{j,t} \equiv 2y^*_{j,t,5y} - (y^*_{j,t,6m} + y^*_{j,t,10y})$.[4] Our relative yield-curve proxies are constructed by taking cross-country differences derived from yield curves estimated on a country-by-country basis, therefore we do not assume any symmetry in the factor structure of yield curves across countries.

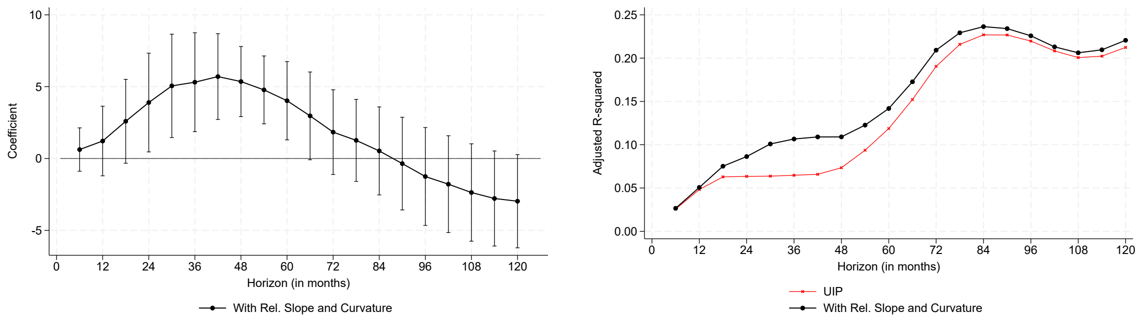
Results. Figure 2a presents our key result, plotting the relative-slope coefficient estimates $\hat{\beta}_{2,\kappa}$ from equation (2). It highlights a tent-shaped relationship across horizons between the relative slope and κ-period exchange-rate dynamics, controlling for interest-rate differentials. Coefficients are significantly different from zero at the 95% level between the 2 to 5-year tenors, business-cycle horizons. The relationship peaks at 3.5 years, where point estimates indicate that a 1pp increase in a country's yield curve slope relative to the US is, on average, associated with a 5.7% exchange-rate depreciation over that horizon and, by equation (3), a commensurate change in *ex post* ERRP.

[4] We prefer these proxies to principal-component estimates of the slope and curvature, which potentially contain look-ahead bias, being defined using weights estimated using information in the whole sample. By construction, our proxies are only based on information available up to time t. Nevertheless, our findings are robust to the use of principal-component-based measures.

Figure 2: Results from augmented UIP regression

(a) Estimated relative-slope coefficients

(b) Adjusted R^2



Notes: (a) Black circles denote $\hat{\beta}_{2,\kappa}$ estimates from regression (2). (b) adjusted R^2 from regression (1) (red crosses) and (2) (black circles). Horizontal axes denote the horizon κ in months. Regressions estimated using pooled monthly data for 6 currencies (AUD, CAD, CHF, EUR, JPY, GBP) against the USD from 1980:01 to 2019:12, including country fixed effects. 95% confidence intervals, calculated using Driscoll and Kraay (1998) standard errors, denoted by black bars around point estimates.

Moreover, at intermediate horizons, the augmented model delivers substantially greater explanatory power. Figure 2b illustrates the associated adjusted R^2. The marginal increase in fit is largest at 2 to 4-year horizons, where the percentage of variation explained by the model exceeds 10%, of which about half is attributable to the inclusion of the relative yield-curve factors. The within-country R^2—tabulated in Appendix B, Table 1—paints an even starker image. At the 3.5-year tenor, the within R^2 from the yield-curve-augmented model is 5.1%, relative just to 0.3% from the baseline UIP regression.

At shorter and longer horizons, especially at tenors of less than 2 years and more than 5, point estimates on the relative yield-curve slope are insignificant and close to zero. Moreover, at all tenors, coefficient estimates on spot-yield differentials $\hat{\beta}_{1,\kappa}$ are insignificantly different from their baseline-regression point estimates, although error bands are somewhat wider.

Robustness. We present robustness analysis for the tent-shaped relationship between currency dynamics across horizons and the relative slope in Appendix B. First, we show the result is robust to the exclusion of the relative yield-curve curvature from regression (2). Second, it is robust to the additional exclusion of spot-yield differentials, which are themselves predicted by the yield-curve slope. Third, but equally important, we demonstrate robustness with alternative, more conservative, inference (Valkanov, 2003; Moon, Rubia, and Valkanov, 2004) and alternative sub-samples (e.g., pre- and post-2008). However, we concede that, like other UIP patterns, the tent-shaped relationship is specific to using the US dollar as the base currency, suggestive of a global 'dollar' factor, consistent with the analysis in, e.g., Jiang (2024).

3 Excess Returns, Risk Premia and the Yield-Curve Slope

The evidence presented in Section 2 highlights a relationship between exchange-rate dynamics, *vis-à-vis* the US dollar, and the relative yield-curve slope at intermediate horizons. In this section, we assess the association between the relative yield-curve slope and different components of bond returns, namely: ERRP and local-currency bond premia. Additionally, this analysis reduces the empirical challenges posed by limiting the number of non-overlapping observations in long-horizon forecasting regressions, like (1) and (2), as κ increases.

3.1 Empirical Setup

Let $P_{t,\kappa}$ denote the price of a κ-maturity zero-coupon bond at time t and $R_{t,\kappa} \geq 1$ denote the gross return on that bond. To decompose bond returns, we distinguish a bond's maturity κ from its holding period h, where $h \leq \kappa$ and $h = \kappa$ if and only if the bond is held to maturity, in which case the analysis coincides with Section 2. The h-month holding period return on a κ-month zero-coupon bond is $HPR^{(\kappa)}_{t,t+h} = P_{t+h,\kappa-h}/P_{t,\kappa}$ (the bond's resale price at $t+h$ when its maturity has diminished by h months relative to its time-t price). The (log) excess return on that bond over the holding period h is thus:

$$rx^{(\kappa)}_{t,t+h} = \log \left[\frac{HPR^{(\kappa)}_{t,t+h}}{R_{t,h}} \right] \tag{4}$$

where $R_{t,h}$ is the gross return on an h-month zero-coupon bond at time t, the risk-free rate.

The h-period (log) return on a Foreign bond, expressed in US dollars, in excess of the risk-free return in the base currency, $rx^{(\kappa),\$}_{t,t+h}$, can be decomposed as:

$$rx^{(\kappa),\$}_{t,t+h} = \log \left[\frac{HPR^{(\kappa)*}_{t,t+h}}{R_{t,h}} \frac{\mathcal{E}_t}{\mathcal{E}_{t+h}} \right] = \log \left[\frac{HPR^{(\kappa)*}_{t,t+h}}{R^*_{t,h}} \right] + \log \left[\frac{R^*_{t,h}}{R_{t,h}} \frac{\mathcal{E}_t}{\mathcal{E}_{t+h}} \right] = rx^{(\kappa)*}_{t,t+h} + rx^{FX}_{t,t+h} \tag{5}$$

where $rx^{(\kappa)*}_{t,t+h}$ represents the (log) local-currency bond return from a Foreign bond and $rx^{FX}_{t,t+h}$ is the (log) currency excess return.

To study the drivers of these returns, we first estimate the following panel regressions for different holding periods h and bond maturities κ:

$$\mathrm{y}^{(\kappa)}_{j,t,h} = \gamma^{(\kappa)}_{2,h} \left(S^*_{j,t} - S_t \right) + f^{(\kappa)}_{j,h} + \varepsilon^{(\kappa)}_{j,t+h} \tag{6}$$

where $\mathrm{y}^{(\kappa)}_{j,t,h}$ is either:

- $rx^{(\kappa),\$}_{j,t,t+h} - rx^{(\kappa)}_{US,t,t+h}$: the *dollar bond-return difference*, the excess return on the Foreign bond in US dollar terms relative to the US return;
- $rx^{FX}_{j,t,t+h}$: the *exchange-rate risk premium (ERRP)*, the excess return from Foreign currency; or,

- $rx^{(\kappa)*}_{j,t,t+h} - rx^{(\kappa)}_{US,t,t+h}$: the *local-currency bond-return difference*, the excess return on the Foreign bond in Foreign-currency terms relative to the US return.

The coefficient $\gamma^{(\kappa)}_{2,h}$ has a similar interpretation to $\beta_{2,\kappa}$ from Section 2 with two differences: the expected sign is now negative, since excess returns are defined on Foreign bonds, as equation (3) clarifies; and the excess returns in our empirical exercises are annualised.

Regression (6) aligns with the specification in Lustig et al. (2019)—although Lustig et al. (2019) consider a 1-month holding period, while we look at 6-month holding periods, so comparison is not exact. Focusing on $h = 1$ and $\kappa = 120$ only, they show that the relative yield-curve slope has an insignificant influence on dollar bond-return differences, but opposing effects on local-currency bond-return differences (positive coefficient) and ERRP (negative coefficient) that cancel out overall. Our empirical framework extends this, assessing the predictability of excess returns with yield-curve slope differentials at a range of maturities κ and holding periods h, bridging the gap between the canonical UIP regressions in Section 2 and Lustig et al. (2019).

To account for the contribution of the relative slope *over and above* spot-yield differentials, as in Section 2, we also estimate the following extending regression:

$$\mathrm{y}^{(\kappa)}_{j,t,h} = \gamma^{(\kappa)}_{1,h}\left(r^*_{j,t,h} - r_{t,h}\right) + \gamma^{(\kappa)}_{2,h}\left(S^*_{j,t} - S_t\right) + f^{(\kappa)}_{j,h} + \varepsilon^{(\kappa)}_{j,t+h} \tag{7}$$

where the maturity of the relative spot yield here matches the holding period h of the excess return on the left-hand side $\mathrm{y}^{(\kappa)}_{j,t,h}$.

3.2 Results

Tables 1 and 2 present the full results for regression (6), with Figure 3 focusing on the coefficient estimates for the 10-year maturity only ($\kappa = 120$). Importantly, where our regression specification most closely matches Lustig et al. (2019), at short holding periods $h = 6$ and the longest maturity $\kappa = 120$, our results mirror theirs. The relative slope is insignificantly associated with the dollar bond-return difference (Panel A), a positive and significant influence on the local-currency bond-return difference (Panel C), and a negative and significant influence on the ERRP (Panel B). The latter two effects approximately cancel out for dollar bond-return differences.[5]

Exploring our results at all holding periods h and for all maturities κ, two observations are noteworthy. First, while the relative yield-curve slope does not significantly predict dollar bond-return differences at the 6-month holding period for 10-year bonds, the relative-slope loading for the same bond maturity is significantly non-zero over some longer holding periods. While, in the former case, the influence of the relative slope on currency and local-currency bond returns offset one another (in line with Lustig et al., 2019), our results indicate that the influence of the relative slope on the currency premium *dominates over longer holding periods*, even for long-term bonds. Nevertheless, for a given holding period, the influence of the relative slope on

[5]More generally, the short-horizon local-currency bond-return difference predictability confirm results for US bond returns (see, e.g., Fama and Bliss, 1987; Campbell and Shiller, 1991; Cochrane and Piazzesi, 2005).

Table 1: Slope coefficient estimates from dollar bond-return and ERRP regressions

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
	Holding Periods h									
	6m	12m	18m	24m	30m	36m	42m	48m	54m	60m
Panel A: Dependent Variable $rx^{(\kappa),\$}_{j,t,t+h} - rx^{(\kappa)}_{US,t,t+h}$										
12m	-2.52***									
	(0.79)									
18m	-2.50***	-2.17***								
	(0.78)	(0.58)								
24m	-2.45***	-2.15***	-2.04***							
	(0.77)	(0.57)	(0.44)							
30m	-2.39***	-2.12***	-2.04***	-1.75***						
	(0.76)	(0.56)	(0.44)	(0.36)						
36m	-2.32***	-2.08***	-2.02***	-1.76***	-1.55***					
	(0.75)	(0.55)	(0.43)	(0.35)	(0.28)					
42m	-2.25***	-2.03***	-1.98***	-1.75***	-1.56***	-1.28***				
	(0.75)	(0.54)	(0.42)	(0.35)	(0.28)	(0.22)				
48m	-2.17***	-1.97***	-1.94***	-1.73***	-1.55***	-1.28***	-0.95***			
	(0.74)	(0.54)	(0.41)	(0.34)	(0.27)	(0.22)	(0.19)			
54m	-2.08***	-1.90***	-1.89***	-1.70***	-1.52***	-1.27***	-0.95***	-0.65***		
	(0.74)	(0.54)	(0.41)	(0.34)	(0.27)	(0.22)	(0.19)	(0.18)		
60m	-1.99***	-1.83***	-1.83***	-1.66***	-1.49***	-1.25***	-0.93***	-0.65***	-0.42**	
	(0.74)	(0.53)	(0.41)	(0.34)	(0.27)	(0.22)	(0.19)	(0.19)	(0.18)	
66m	-1.89**	-1.75***	-1.77***	-1.62***	-1.46***	-1.22***	-0.91***	-0.63***	-0.41**	-0.30*
	(0.74)	(0.53)	(0.40)	(0.33)	(0.27)	(0.22)	(0.19)	(0.18)	(0.18)	(0.17)
72m	-1.80**	-1.68***	-1.71***	-1.57***	-1.41***	-1.18***	-0.88***	-0.61***	-0.40**	-0.29*
	(0.74)	(0.53)	(0.40)	(0.33)	(0.27)	(0.22)	(0.19)	(0.18)	(0.17)	(0.17)
78m	-1.70**	-1.60***	-1.64***	-1.51***	-1.36***	-1.14***	-0.84***	-0.58***	-0.38**	-0.28*
	(0.74)	(0.53)	(0.40)	(0.33)	(0.27)	(0.22)	(0.19)	(0.18)	(0.17)	(0.16)
84m	-1.61**	-1.52***	-1.58***	-1.46***	-1.31***	-1.10***	-0.81***	-0.55***	-0.36**	-0.26
	(0.74)	(0.53)	(0.40)	(0.33)	(0.27)	(0.22)	(0.19)	(0.18)	(0.17)	(0.16)
90m	-1.51**	-1.44***	-1.51***	-1.40***	-1.26***	-1.05***	-0.76***	-0.52***	-0.33**	-0.24
	(0.74)	(0.53)	(0.40)	(0.33)	(0.27)	(0.22)	(0.19)	(0.18)	(0.17)	(0.16)
96m	-1.42*	-1.36**	-1.44***	-1.34***	-1.21***	-1.00***	-0.72***	-0.49***	-0.31*	-0.22
	(0.74)	(0.53)	(0.40)	(0.33)	(0.27)	(0.22)	(0.19)	(0.18)	(0.16)	(0.15)
102m	-1.33*	-1.29**	-1.37***	-1.29***	-1.16***	-0.95***	-0.68***	-0.46**	-0.28*	-0.20
	(0.74)	(0.53)	(0.40)	(0.33)	(0.27)	(0.22)	(0.19)	(0.18)	(0.16)	(0.15)
108m	-1.24*	-1.21**	-1.31***	-1.23***	-1.10***	-0.91***	-0.64***	-0.42**	-0.25	-0.18
	(0.74)	(0.53)	(0.40)	(0.33)	(0.27)	(0.22)	(0.20)	(0.18)	(0.16)	(0.15)
114m	-1.15	-1.14**	-1.24***	-1.17***	-1.05***	-0.86***	-0.59***	-0.39**	-0.22	-0.16
	(0.74)	(0.54)	(0.40)	(0.33)	(0.27)	(0.22)	(0.20)	(0.18)	(0.16)	(0.15)
120m	-1.07	-1.06**	-1.18***	-1.12***	-1.00***	-0.81***	-0.55***	-0.35*	-0.20	-0.14
	(0.74)	(0.54)	(0.40)	(0.33)	(0.27)	(0.23)	(0.20)	(0.18)	(0.16)	(0.14)
Panel B: Dependent Variable $rx^{FX}_{j,t,t+h}$										
S^R	-2.51***	-2.16***	-2.03***	-1.73***	-1.53***	-1.27***	-0.95***	-0.65***	-0.43**	-0.31*
	(0.81)	(0.59)	(0.46)	(0.37)	(0.29)	(0.23)	(0.19)	(0.19)	(0.18)	(0.18)

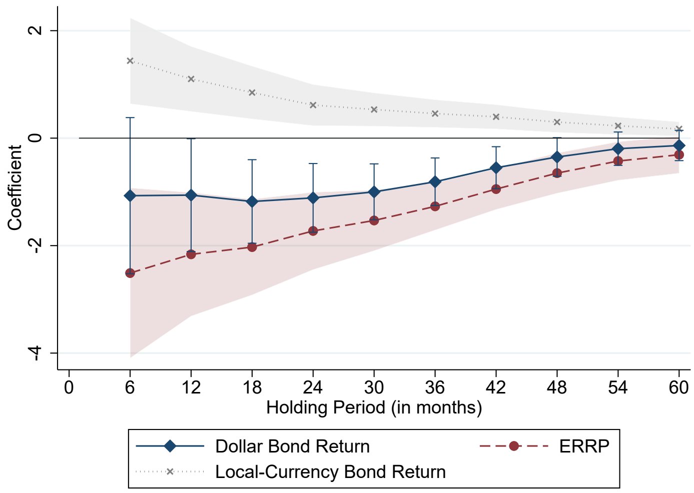
Notes: Coefficient estimates on the relative yield curve slope $S^R_t \equiv S^*_t - S_t$ from regressions with the (log) dollar bond-return difference (Panel A) or the (log) ERRP (Panel B) as dependent variables. Regressions estimated using pooled end-of-month data for 6 currencies (AUD, CAD, CHF, EUR, JPY, GBP) against the USD for 1980:01-2019:12. Log returns are annualised. All regressions include country fixed effects. The panels are unbalanced and Driscoll and Kraay (1998) standard errors are reported in parentheses. $*$, $**$ and $***$ denote significant point estimates at 10%, 5% and 1% levels, respectively.

Table 2: Slope coefficient estimates from local-currency bond-return regressions

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
	Holding Periods h									
	6m	12m	18m	24m	30m	36m	42m	48m	54m	60m
Panel C: Dependent Variable $rx^{(\kappa)}_{j,t,t+h} - rx^{(\kappa)}_{US,t,t+h}$										
12m	-0.01									
	(0.05)									
18m	0.02	-0.01								
	(0.09)	(0.03)								
24m	0.07	0.01	-0.01							
	(0.12)	(0.06)	(0.03)							
30m	0.12	0.04	-0.01	-0.03						
	(0.16)	(0.09)	(0.05)	(0.02)						
36m	0.19	0.08	0.01	-0.03	-0.02					
	(0.19)	(0.12)	(0.08)	(0.04)	(0.02)					
42m	0.26	0.14	0.05	-0.03	-0.02	-0.01				
	(0.21)	(0.14)	(0.10)	(0.06)	(0.03)	(0.01)				
48m	0.35	0.20	0.09	-0.01	-0.01	-0.01	-0.01			
	(0.24)	(0.16)	(0.11)	(0.07)	(0.05)	(0.03)	(0.01)			
54m	0.43*	0.26	0.14	0.03	0.01	-0.00	-0.00	-0.00		
	(0.26)	(0.17)	(0.13)	(0.09)	(0.06)	(0.04)	(0.02)	(0.01)		
60m	0.52*	0.33*	0.20	0.06	0.04	0.02	0.02	0.01	0.00	
	(0.28)	(0.19)	(0.14)	(0.10)	(0.07)	(0.05)	(0.04)	(0.02)	(0.01)	
66m	0.62**	0.41**	0.26	0.11	0.08	0.05	0.04	0.02	0.01	0.01
	(0.30)	(0.20)	(0.16)	(0.11)	(0.09)	(0.06)	(0.05)	(0.03)	(0.02)	(0.01)
72m	0.71**	0.49**	0.32*	0.16	0.12	0.09	0.07	0.04	0.03	0.02
	(0.31)	(0.21)	(0.17)	(0.12)	(0.09)	(0.07)	(0.05)	(0.04)	(0.03)	(0.02)
78m	0.81**	0.56**	0.39**	0.21	0.17*	0.13*	0.10*	0.07	0.05	0.03
	(0.33)	(0.23)	(0.18)	(0.13)	(0.10)	(0.08)	(0.06)	(0.05)	(0.04)	(0.02)
84m	0.90***	0.64***	0.45**	0.27*	0.22**	0.17**	0.14**	0.10*	0.07	0.05
	(0.34)	(0.24)	(0.19)	(0.14)	(0.11)	(0.09)	(0.07)	(0.06)	(0.04)	(0.03)
90m	1.00***	0.72***	0.52***	0.33**	0.27**	0.22**	0.18**	0.13**	0.09*	0.07*
	(0.35)	(0.25)	(0.20)	(0.15)	(0.12)	(0.09)	(0.08)	(0.06)	(0.05)	(0.04)
96m	1.09***	0.80***	0.59***	0.38**	0.32**	0.27***	0.23***	0.16**	0.12**	0.09*
	(0.36)	(0.26)	(0.21)	(0.16)	(0.13)	(0.10)	(0.09)	(0.07)	(0.06)	(0.04)
102m	1.18***	0.88***	0.65***	0.44***	0.38***	0.32***	0.27***	0.20**	0.15**	0.11**
	(0.37)	(0.27)	(0.22)	(0.17)	(0.14)	(0.11)	(0.09)	(0.08)	(0.07)	(0.05)
108m	1.27***	0.95***	0.72***	0.50***	0.43***	0.36***	0.31***	0.23***	0.17**	0.13**
	(0.39)	(0.29)	(0.23)	(0.18)	(0.14)	(0.12)	(0.10)	(0.09)	(0.07)	(0.06)
114m	1.36***	1.03***	0.79***	0.56***	0.48***	0.41***	0.36***	0.27***	0.20***	0.15**
	(0.40)	(0.30)	(0.24)	(0.19)	(0.15)	(0.12)	(0.11)	(0.09)	(0.08)	(0.06)
120m	1.44***	1.10***	0.85***	0.62***	0.53***	0.46***	0.40***	0.30***	0.23***	0.17**
	(0.41)	(0.31)	(0.25)	(0.20)	(0.16)	(0.13)	(0.12)	(0.10)	(0.08)	(0.07)

Notes: Coefficient estimates on the relative yield curve slope $S^*_t - S_t$ from regressions with the (log) local-currency bond-return difference (Panel C) as dependent variable. Regressions estimated using pooled end-of-month data for 6 currencies (AUD, CAD, CHF, EUR, JPY, GBP) against the USD for 1980:01-2019:12. Log returns are annualised. All regressions include country fixed effects. The panels are unbalanced and Driscoll and Kraay (1998) standard errors are reported in parentheses. $*$, $**$ and $***$ denote significant point estimates at 10%, 5% and 1% levels, respectively.

Figure 3: Estimated relative slope coefficients from excess-return regressions across holding periods for 10-year maturity



Notes: $\hat{\gamma}_{2,h}^{(120)}$ estimates from regression (6) for dollar bond-return differences (blue diamonds), exchange-rate risk premia (maroon circles) and local-currency bond-return differences (grey crosses). Horizontal axis denotes the holding period h in months. Regressions estimated using pooled monthly data for 6 currencies (AUD, CAD, CHF, EUR, JPY, GBP) against the USD from 1980:01 to 2019:12, including country fixed effects. 95% confidence intervals, calculated using Driscoll and Kraay (1998) standard errors, denoted by shaded areas/bars around point estimates.

dollar bond returns decreases in magnitude with maturity.

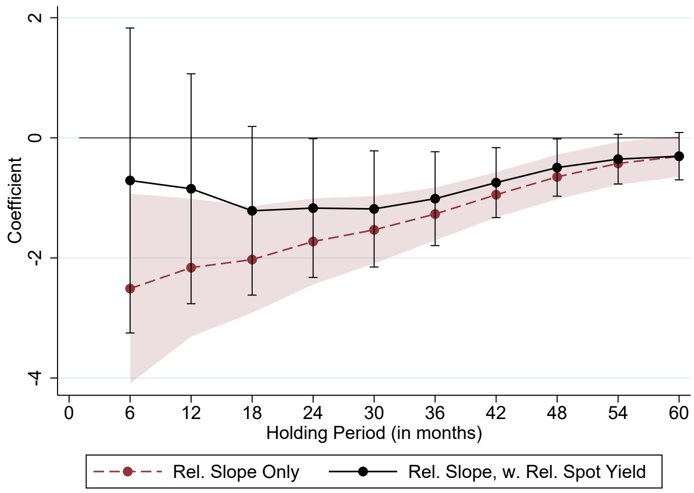
Second, for a given maturity, the loading on the relative slope tends to peak in magnitude at short-to-medium holding periods for the dollar bond risk premium. For the 102-month maturity, and above, the peak coefficient occurs at the 18-month holding period. This gives rise to a(n inverse) tent shaped relationship in the coefficients across h, as the Figure 3 shows for dollar bond-return differences for the 10-year maturity. Although significant at shorter holding periods and longer maturities, the relative slope loadings are quantitatively small for local-currency bond premia and are dominated by loadings on currency excess returns in explaining the relative slope's impact on relative dollar-bond risk premia.

The (inverse) tent-shaped relationship between the relative slope and excess returns arises when controlling for spot-yield differentials, as in regression (7). Figure 4 demonstrates this, presenting the coefficient estimates on the relative slope for ERRP from that regression, compared to the benchmark regression (6). The coefficient estimates are also tabulated in Panel A of Table 3. In the specification with spot-yield differentials as controls, the negative coefficient on the relative slope is significantly different from zero from the 2 to 4-year holding periods. Therefore our results indicate that the relative slope has predictive power over and above spot-yield differences at business-cycle horizons specifically.

3.3 Robustness

In this sub-section, we briefly summarize the robustness of these empirical findings.

Figure 4: Estimated relative slope coefficients from ERRP regressions across holding periods with and without controlling for relative spot-yield differentials



Notes: $\hat{\gamma}_{2,h}$ estimates from regressions (6) (maroon circles) and (7) (black circles) for exchange-rate risk premia. Horizontal axis denotes the holding period h in months. Regressions estimated using pooled monthly data for 6 currencies (AUD, CAD, CHF, EUR, JPY, GBP) against the USD from 1980:01 to 2019:12, including country fixed effects. 95% confidence intervals, calculated using Driscoll and Kraay (1998) standard errors, denoted by shaded areas/bars around point estimates.

Sub-Sample Stability. Panels B, C and D of Table 3 demonstrate that the association between the relative slope and ERRP, against the US dollar, is robust to sub-sample splits. Panel B presents a pre-global financial crisis sample (1980:01-2008:06), Panel C a shorter pre-crisis sample (1990:01-2008:06), and Panel D shows results from a sample spanning the period after the crisis (1990:01-2019:12).

Liquidity Yields. We also document that the significant relationship between the relative slope and ERRP at business-cycle horizons is robust to controlling for liquidity (or convenience) yields (i.e., non-pecuniary returns), which recent contributions to the literature have emphasised a role for in exchange rate determination (see, e.g., Engel and Wu, 2022; Jiang et al., 2021). To do this, we use data on the term structure of liquidity yields from Du et al. (2018). These measure the difference between riskless market rates and government yields at different maturities to quantify the implicit yield on a government bond, correcting for other frictions in forward markets and sovereign risk. Let $\eta^R_{j,t,\kappa}$ denote the κ-horizon liquidity premium for a κ-horizon US government bond relative to an equivalent-maturity Foreign government bond yield in country j. An increase in $\eta^R_{j,t,\kappa}$ reflects an increase in the relative liquidity of US Treasuries *vis-à-vis* country j. With these measures, we extend regression (6) by estimating:[6]

$$\mathrm{y}^{(\kappa)}_{j,t,h} = \gamma^{(\kappa)}_{1,h}\left(S^*_{j,t} - S_t\right) + \gamma^{(\kappa)}_{2,h}\eta_{j,t,\kappa} + f^{(\kappa)}_{j,h} + \varepsilon^{(\kappa)}_{j,t+h} \tag{8}$$

[6] Although the Du et al. (2018) data is available from 1991:04 for some countries and tenors, some series begin as late as 1999:01 due to data availability.

Table 3: Robustness of relative slope coefficient estimates from regression (6) for $rx^{FX}_{t,t+h}$

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
					Holding Periods h					
	6m	12m	18m	24m	30m	36m	42m	48m	54m	60m
A: Controlling for interest-rate differentials										
S^R	-0.71	-0.85	-1.22*	-1.17**	-1.18**	-1.01**	-0.75**	-0.50**	-0.36*	-0.31
	(1.30)	(0.98)	(0.72)	(0.59)	(0.49)	(0.40)	(0.30)	(0.24)	(0.21)	(0.20)
r^R_h	3.19*	1.31*	0.61	0.36	0.20	0.14	0.11	0.08	0.04	0.00
	(1.90)	(0.75)	(0.40)	(0.27)	(0.20)	(0.15)	(0.11)	(0.08)	(0.06)	(0.05)
B: 1980:01-2008:06 sub-sample										
S^R	-2.92***	-2.39***	-2.26***	-1.95***	-1.73***	-1.48***	-1.18***	-0.89***	-0.66***	-0.54***
	(0.86)	(0.65)	(0.51)	(0.41)	(0.31)	(0.24)	(0.20)	(0.19)	(0.19)	(0.20)
C: 1990:01-2008:06 sub-sample										
S^R	-2.00**	-1.66**	-1.68***	-1.49***	-1.34***	-1.16***	-0.92***	-0.67***	-0.48**	-0.35*
	(0.92)	(0.66)	(0.51)	(0.40)	(0.29)	(0.21)	(0.17)	(0.17)	(0.19)	(0.20)
D: 1990:01-2019:12 sub-sample										
S^R	-1.71*	-1.55**	-1.53***	-1.32***	-1.17***	-0.96***	-0.68***	-0.41**	-0.23	-0.09
	(0.88)	(0.63)	(0.49)	(0.39)	(0.29)	(0.22)	(0.19)	(0.19)	(0.19)	(0.18)
E: Controlling for liquidity yields										
S^R	-1.81	-1.80*	-1.77**	-1.47**	-1.48***	-1.37***	-1.02***	-0.65*	-0.38	-0.15
	(1.60)	(1.08)	(0.72)	(0.60)	(0.48)	(0.39)	(0.37)	(0.39)	(0.38)	(0.35)
η^R_{10y}	0.07*	0.07**	0.06**	0.07***	0.07***	0.07***	0.07***	0.07***	0.07***	0.07***
	(0.04)	(0.03)	(0.02)	(0.02)	(0.02)	(0.02)	(0.01)	(0.01)	(0.01)	(0.01)

Notes: Coefficient estimates on the relative yield curve slope $S^R_t \equiv S^* - S$ from regressions with the (log) ERRP as dependent variable. Regressions estimated using pooled end-of-month data for 6 currencies (AUD, CAD, CHF, EUR, JPY, GBP) against the USD. Log returns are annualised. All regressions include country fixed effects. The panel is unbalanced and Driscoll and Kraay (1998) standard errors are reported in parentheses. $*$, $**$ and $***$ denote significant point estimates at 10%, 5% and 1% levels, respectively. Regressions in Panel A additionally include spot-yield differentials $r^R \equiv r^*_h - r_h$ as regressors. Regressions in Panel E additionally control for 10-year liquidity yields η^R_{10y} as regressors.

where $y^{(\kappa)}_{j,t,h}$ has the same definition as in regression (6) and $\gamma^{(\kappa)}_{2,h}$ can be interpreted as the average influence of a 1pp increase in relative US Treasury convenience. When the ERRP is the dependent variable, we expect $\gamma^{(\kappa)}_{2,h}$ to be positive.

The results for current returns $rx^{FX}_{t,t+h}$ are shown for the 10-year liquidity yield η^R_{10y} in Panel E of Table 3. As before, the relative slope coefficient is significantly associated with ERRP at business-cycle holding periods—here 1 to 4 years. Corresponding investigation into the dollar bond-return differences confirms that the influence of the relative slope on dollar bond returns predominantly works through ERRP.

Strikingly, the $\gamma^{(\kappa)}_{2,h}$ coefficients reveal a stronger association between liquidity yields and ERRP at longer horizons. The coefficients on the relative liquidity yield rise monotonically with respect to holding periods and grow in significance. This stands in contrast to most existing studies into liquidity yields and exchange-rate dynamics (e.g., Engel and Wu, 2022; Jiang et al., 2021), which have focused on short-horizon returns.

Cross-Sectional Returns. To account for returns in the cross-section, we also consider the average returns across maturities κ and holding periods h from a simple investment strategy based on the yield-curve slope. Specifically, we consider a strategy that goes long the Foreign bond and short the US bond when the Foreign yield curve is less steep than the US one, and *vice versa*. The results are presented in Appendix C. They demonstrate that average returns have a tent-shaped pattern across holding periods, for different maturities, supporting evidence of the yield curve slope's predictive role for returns.

4 What Explains Predictability By The Relative Slope?

Having documented a robust association between the relative yield-curve slope and ERRP, we draw on theory to explain their relationship.

4.1 Preference-Free Setup

Consider a model with two countries: Home (base currency, i.e., US) and Foreign (denoted by an *), each populated by representative investor—which we later generalize in Section 5.3.

Pricing Kernels and Stochastic Discount Factors. The Home nominal pricing kernel V_t represents the marginal value of a currency unit at time t. The absence of arbitrage implies the existence of a nominal SDF $M_{t,t+\kappa}$, which is given by the growth rate of the pricing kernel between periods t and $t+\kappa$: $M_{t,t+\kappa} = V_{t+\kappa}/V_t$.

We assume that investors can trade freely in Home- and Foreign-currency denominated risk-free bonds across maturities. The price of a Home zero-coupon bond that promises one currency unit κ periods into the future is given by: $P_{t,\kappa} = \mathbb{E}_t\left[M_{t,t+\kappa}\right] = \mathbb{E}_t\left[M_{t,t+1}P_{t+1,\kappa-1}\right]$, where $M_{t,t+1}$ denotes the one-period SDF and, by recursive substitution, $M_{t,t+\kappa} \equiv \prod_{i=0}^{\kappa-1} M_{t+i,t+i+1}$. Defining the gross return on a Home κ-period zero-coupon bond as $R_{t,\kappa} \equiv 1/P_{t,\kappa} \equiv (1+r_{t,\kappa}) \geq 1$, then:

$$\frac{1}{R_{t,\kappa}} = \mathbb{E}_t\left[M_{t,t+\kappa}\right] \tag{9}$$

which can be expanded as:

$$-r_{t,\kappa} = \mathbb{E}_t[m_{t,t+\kappa}] + \mathcal{L}_t(M_{t,t+\kappa}), \tag{10}$$

where $m_{t,t+\kappa} = \ln M_{t,t+\kappa}$ and $\mathcal{L}_t\left(M_{t,t+\kappa}\right) = \ln \mathbb{E}_t[M_{t,t+\kappa}] - \mathbb{E}_t[m_{t,t+\kappa}]$ denotes the conditional multi-period entropy of the SDF.[7] Foreign expressions are analogously derived.

Exchange Rates and Currency Risk Premia. The exchange rate $\mathcal{E}_t$ is defined as the Foreign price of a unit of Home currency such that an increase corresponds to a Foreign depreciation. When engaging in cross-border asset trade, the Euler equation for a Home investor holding a κ-period Foreign currency-denominated bond is:

$$1 = \mathbb{E}_t\left[M_{t,t+\kappa}\frac{\mathcal{E}_t}{\mathcal{E}_{t+\kappa}}R^*_{t,\kappa}\right] \tag{11}$$

[7] If we assume one-period SDFs, $M^{(*)}_{t,t+1}$ are log-normally distributed, then (10) evaluated at $\kappa = 1$ is equivalent to: $-r_t = \mathbb{E}_t[m_{t,t+1}] + \frac{1}{2}\text{var}_t\left(m_{t,t+1}\right)$. However, multi-period SDFs ($\kappa > 1$) will generally not be log-normally distributed if risk is heteroskedastic.

By no-arbitrage, the change in the nominal exchange rate corresponds to the ratio of SDFs:

$$\frac{\mathcal{E}_{t+\kappa}}{\mathcal{E}_t} = \frac{M_{t,t+\kappa}}{M^*_{t,t+\kappa}} e^{\eta_{t,t+\kappa}} \tag{12}$$

for all $\kappa > 0$, where $\eta_{t,t+\kappa}$ is the log incomplete-markets wedge as defined in Backus et al. (2001), such that $\eta_{t,t+\kappa} = 0$ characterizes complete markets.

The (log) κ-period *ex-ante* currency risk premium $\mathbb{E}_t[rx^{FX}_{t,t+\kappa}]$ can be written as the difference in entropy of the Home and Foreign SDFs:

$$\begin{aligned}\frac{1}{\kappa}\mathbb{E}_t\left[rx^{FX}_{t,t+\kappa}\right] &= \frac{1}{\kappa}\left(r^*_{t,\kappa} - r_{t,\kappa} - \mathbb{E}_t\left[\Delta^{\kappa} e_{t+\kappa}\right] + \mathbb{E}_t[\eta_{t,t+\kappa}]\right) \\ &= \frac{1}{\kappa}\left(\mathcal{L}_t\left(M_{t,t+\kappa}\right) - \mathcal{L}_t\left(M^*_{t,t+\kappa}\right) + \mathbb{E}_t[\eta_{t,t+\kappa}]\right)\end{aligned} \tag{13}$$

This is the multi-period generalization of the standard one-period UIP return (e.g., Engel, 2016).

Transitory-Permanent Risk Decomposition. As a first pass, to assess the nature of risks driving ERRP and the yield-curve slope, we use the Alvarez and Jermann (2005) decomposition of the pricing kernel V_t into a permanent component $V_t^{\mathbb{P}}$ and a transitory component $V_t^{\mathbb{T}}$:

$$V_t = V_t^{\mathbb{P}} V_t^{\mathbb{T}}, \quad \text{where } V_t^{\mathbb{T}} = \lim_{\kappa\to\infty} \frac{\delta^{t+\kappa}}{P_{t,\kappa}} \tag{14}$$

where the constant δ is chosen to satisfy the regularity condition: $0 < \lim_{\kappa\to\infty} P_{t,\kappa}/\delta^{\kappa} < \infty$ for all t. A pricing kernel V_t is defined as having only transitory innovations if $\lim_{\kappa\to\infty} \frac{\mathbb{E}_{t+1}[V_{t+\kappa}]}{\mathbb{E}_t[V_{t+\kappa}]} = 1$. So, its permanent component follows a martingale, defined by: $V_t^{\mathbb{P}} = \lim_{\kappa\to\infty} \frac{\mathbb{E}_t[V_{t+\kappa}]}{\delta^{t+\kappa}}$.

The key result is that the return on an infinite-maturity bond can be written as a function of transitory innovations to SDFs only: $R_{t,\infty} = \lim_{\kappa\to\infty} R_{t,\kappa} = V_t^{\mathbb{T}}/V_{t+1}^{\mathbb{T}} = 1/M^{\mathbb{T}}_{t,t+1} = \exp(-m^{\mathbb{T}}_{t,t+1})$, where $m^{\mathbb{T}}_{t,t+1}$ denotes the transitory component of the SDF. In contrast, one-period bond returns, defined by equation (9), depend on both transitory and permanent innovations to SDFs.

Returning to currency premia, the failure to reject long-horizon UIP requires equation (13) to be approximately zero as $\kappa \to \infty$ and this implies the equalization of the entropy of permanent SDF components since $\lim_{\kappa\to\infty} \frac{1}{\kappa} M_{t,t+\kappa} = V^{\mathbb{P}}_{t+1}/V^{\mathbb{P}}_t = M^{\mathbb{P}}_{t,t+1}$ (see Lustig et al., 2019).[8] Consequently, short and medium-horizon ERRP must reflect cross-country differences in the volatility of transitory innovations to SDFs.

$$\frac{1}{\kappa}\mathbb{E}_t\left[rx^{FX}_{t,t+\kappa}\right] \approx \frac{1}{\kappa}\left(\mathcal{L}_t\left(M^{\mathbb{T}}_{t,t+\kappa}\right) - \mathcal{L}_t\left(M^{\mathbb{T}\,*}_{t,t+\kappa}\right)\right) \tag{15}$$

[8] Alvarez and Jermann (2005) emphasize that to jointly rationalize high equity premia and low bond premia, most SDF volatility must arise from permanent SDF innovations. The contrast in exchange-rate markets may arise due to horizon-varying risk transmission across countries.

4.2 Yield-Curve Slope and Transitory Risk

As well as playing a role in short-to-medium-horizon ERRP, transitory risk is reflected in the yield-curve slope. Define the (log) excess return from buying a n-period Home bond at time t for price $P_{t,n} = 1/R_{t,n}$ and selling it at time $t+1$ for $P_{t+1,n-1} = 1/R_{t+1,n-1}$ as $rx^{(n)}_{t,t+1} = p_{t+1,n-1} - p_{t,n} - y_{t,1}$, where $p_{t,n} \equiv \log(P_{t,n})$ and $y_{t,n} \equiv -\frac{1}{n}p_{t,n} \equiv \frac{1}{n}r_{t,n}$ is the annualised yield on a n-period bond. Assuming, for convenience, SDFs and prices are jointly log-normally distributed, this excess return can be written as:

$$\mathbb{E}_t\left[rx^{(n)}_{t,t+1}\right] + \frac{1}{2}\mathrm{var}_t\left(r_{t+1,n}\right) = -\mathrm{cov}_t\left(m^{\mathbb{T}}_{t,t+1}, \mathbb{E}_{t+1}\sum_{i=1}^{n-1} m^{\mathbb{T}}_{t+i,t+i+1}\right) \tag{16}$$

Over long enough samples, this risk premium is approximately equal to the yield-curve slope, $\mathbb{E}_t[rx^{(n)}_{t,t+1}] \approx S_t$ where $S_t \equiv y_{t,n} - y_{t,1}$, implying that the yield curve will be upward sloping on average if the covariance term is negative (Piazzesi and Schneider, 2007).[9]

Two key implications follow from (16). First, the bond premium, and therefore the yield-curve slope, only captures transitory innovations to investors' SDFs. The autocovariance of SDFs will be zero for permanent SDF innovations. Second, because the autocovariance must be negative to generate yield curves that slope upwards on average, the premium and slope reflect the time dependence of SDFs. The bond premium is positive if today's one-period SDF is negatively correlated with expected future marginal utility, consistent with a notion of transitory 'business-cycle' risk. That is, if households receive relatively good news about the distant future, they expect to value consumption less at long horizons (i.e., lower $\mathbb{E}_t[m_{t+i,t+i+1}]$ for some $i > 0$), but relatively highly in the near term (i.e., higher $m_{t,t+1}$). Therefore, the relative yield-curve slope $S^*_t - S_t$ can be understood to capture asymmetry or asynchronicity in business-cycle risk across countries.

4.3 Empirical Links Between Macro Expectations, Relative Slope and ERRP

Next, we use macro survey data to investigate the extent to which asymmetries in business-cycle expectations drive the relationship between the relative yield-curve slope and ERRP.

4.3.1 Expectations and the Yield-Curve Slope

The links between *country-specific* yield-curve slopes and macroeconomic outcomes have been widely studied (e.g., Estrella and Hardouvelis, 1991; Estrella and Mishkin, 1998; Estrella, 2005). Here, we investigate the relationship between *relative* yield-curve slopes across countries and *relative* business-cycle expectations using data from professional forecasters working at large financial institutions from *Consensus Economics*.

Specifically, we use forecasters expectations for GDP growth and inflation in each country

[9] To derive this, re-write the excess return $rx^{(n)}_{t,t+1}$ as: $p_{t+1,n-1} - p_{t,n} - y_{t,1} = ny_{t,n} - (n-1)y_{t+1,n-1} - y_{t,1} = y_{t,n} - y_{t,1} - (n-1)(y_{t+1,n-1} - y_{t,n})$. Over a long enough sample and with large n, the difference between the average $(n-1)$-period yield and the average n-period yield is zero, implying that $\mathbb{E}_t[rx^{(n)}_{t,t+1}] \approx y_{t,n} - y_{t,1} \equiv S_t$.

Table 4: Association between relative yield curve slope and relative business-cycle expectations

	(1)	(2)	(3)	(4)
$\overline{gdp}^{0,e}$	-0.4006***		-0.3204***	-0.2446***
	(0.0910)		(0.0880)	(0.0753)
$\overline{gdp}^{1,e}$	0.5734***		0.4172***	0.3715***
	(0.1566)		(0.1606)	(0.1396)
$\overline{cpi}^{0,e}$		-0.3865***	-0.3828***	-0.2709***
		(0.0984)	(0.1062)	(0.0940)
$\overline{cpi}^{1,e}$		-0.4413***	-0.2896**	-0.2962**
		(0.1326)	(0.1245)	(0.1329)
std($gdp^{0,e}$)				-0.3530
				(0.4798)
std($gdp^{1,e}$)				0.3086
				(0.4192)
std($cpi^{0,e}$)				0.9720*
				(0.4953)
std($cpi^{1,e}$)				1.0374**
				(0.4097)
Constant	-0.6450***	-1.0956***	-1.0307***	-0.8349***
	(0.1651)	(0.1469)	(0.1801)	(0.1572)
# Countries	6	6	6	6
Country FE	YES	YES	YES	YES
Within R^2	0.107	0.159	0.220	0.216

Notes: Coefficient estimates from variants of regression (17), with the relative yield curve slope as the dependent variable. Regressions estimated using pooled end-of-month data for 6 countries (with currencies: AUD, CAD, CHF, EUR, JPY, GBP) against the US. All regressions include country fixed effects. The panel is unbalanced and Driscoll and Kraay (1998) standard errors are reported in parentheses. $*$, $**$ and $***$ denote significant point estimates at 10%, 5% and 1% levels, respectively.

for the period over which data is available for all G7 economies (1990:01-2019:12). The forecasts are formed for the current year ($y = 0$) and the next year ($y = 1$). We denote the average expectations of country-j forecasters for year-y GDP and inflation by $\overline{gdp}^{y,e}_{j,t}$ and $\overline{\pi}^{y,e}_{j,t}$, respectively. We also use data capturing uncertainty around forecasts, labelling the cross-sectional standard deviation of GDP and inflation expectations, across forecasters, by $\text{std}(gdp^{y,e})_{j,t}$ and $\text{std}(\pi^{y,e})_{j,t}$, respectively.

We illustrate the link between relative business-cycle expectations and the relative yield curve slope by estimating variants of the following regression:

$$\begin{aligned} S^*_{j,t} - S_t = \sum_{y=0,1} \Big[& \vartheta_1 \left(\overline{gdp}^{y,e*}_{j,t} - \overline{gdp}^{y,e}_{US,t}\right) + \vartheta_2 \left(\overline{\pi}^{y,e*}_{j,t} - \overline{\pi}^{y,e}_{US,t}\right) \\ & + \vartheta_3 \left(\text{std}(gdp^{y,e*})_{j,t} - \text{std}(gdp^{y,e})_{US,t}\right) + \vartheta_4 \left(\text{std}(\pi^{y,e*})_{j,t} - \text{std}(\pi^{y,e})_{US,t}\right) \Big] \\ & + f_j + \epsilon_{j,t} \end{aligned} \tag{17}$$

Table 4 presents the estimated coefficients. The coefficients on average expectations for GDP and inflation, for the current and next year, are strongly significant in all specifications. The mean GDP-expectation coefficient changes sign across horizon, reflecting business-cycle dynamics. The coefficient on the current-year expectation indicates that relatively high near-term GDP-growth expectations are associated with a relatively flat yield curve—consistent with higher short-term rates in booms. In contrast, relatively high expectations for future GDP growth are associated with a relatively steep yield curve—consistent with expectations of higher short-term interest rates in the future. The coefficients on mean inflation expectations are negative at both horizons, possibly indicating lower-frequency dynamics through price expectations: relatively high inflation expectations are associated with a relatively flat yield curve, consistent with a need for higher short-term interest rates to stave off persistence in the rate of price increases. In addition, column (4) highlights some role for uncertainty about inflation in yield curve slopes, with relatively high uncertainty associated with a relatively steep yield curve slope—a feature that is consistent with inflation risk leading nominal bonds to command a term premium. All in all, the four specifications demonstrate a strong association between asymmetries in business-cycle expectations and the relative yield-curve slope across countries, which is also consistent with theory.

4.3.2 Expectations and Excess Currency Returns

To reconnect movements in exchange rates to fundamentals, we test whether the component of the relative slope explained by business-cycle expectations is the main driver of ERRP dynamics at business-cycle horizons. To do so, we recover fitted values $\hat{S}^R_{j,t} \equiv \widehat{S^*_{j,t} - S_t}$ and residuals $\hat{\epsilon}_{j,t}$ from estimates of equation (17). We construct the fitted values and residuals by estimating equation (17) on a country-by-country basis, in order to account for cross-country heterogeneity. Pooling these estimates across countries, we then estimate variants of the following extension to regressions (6) and (7):

$$\mathrm{y}^{(\kappa)}_{j,t,h} = \gamma^{(\kappa)}_{1,h}\left(r^*_{j,t,h} - r_{t,h}\right) + \gamma^{(\kappa)}_{2,h}\hat{S}^R_{j,t} + \gamma^{(\kappa)}_{3,h}\hat{\epsilon}_{j,t} + f^{(\kappa)}_{j,h} + \varepsilon^{(\kappa)}_{j,t+h} \tag{18}$$

where we replace the observed relative slope $S^*_{j,t} - S_t$ with the fitted value $\hat{S}^R_{j,t}$ and additionally include the residual $\hat{\epsilon}_{j,t}$, which captures the component of the relative slope which is unexplained by variation in macroeconomic expectations.[10]

Table 5 presents the results from the regression above excluding spot-yield differentials (Panel A) and including them (Panel B). In either case, the coefficient on the fitted relative slope is significantly negative across similar holding periods to the baseline regressions involving observed relative yield-curve slopes(6) and (7). Panel B can be understood as a spanning regression, in the spirit of Joslin et al. (2014) indicating that the component of the relative slope explained by cross-country asymmetries in macroeconomic expectations has explanatory power for ERRP, orthogonal to interest rates.

Figure 5 plots the $\gamma_{2,h}$ and $\gamma_{3,h}$ estimates from the regression that includes the spot-yield differential. While the coefficient estimates on the residual are insignificant across holding peri-

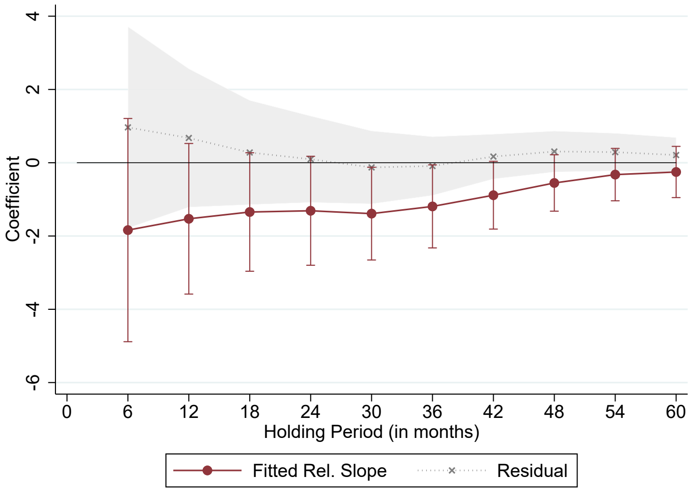
[10]The inclusion of the fitted residual $\hat{\epsilon}$ alongside the fitted value $\hat{S}^R$ from regression (17) additionally deals with concerns about inference with generated regressors. Pagan (1984) shows that consistent inference is possible with generated regressors when fitted values and residuals are used together in the same regression specification.

Table 5: Estimated relationship between ERRP and the component of relative slope driven by business-cycle expectations

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
	Holding Periods h									
	6m	12m	18m	24m	30m	36m	42m	48m	54m	60m
A: Relative yield-curve slope terms only										
$\hat{S}^R$	-3.67***	-3.13***	-2.61***	-2.22***	-1.96***	-1.60***	-1.26***	-0.82**	-0.47	-0.28
	(1.27)	(0.99)	(0.80)	(0.76)	(0.66)	(0.56)	(0.45)	(0.38)	(0.33)	(0.32)
$\hat{\epsilon}$	-1.05	-1.09	-1.12**	-0.90**	-0.73**	-0.51**	-0.20	0.03	0.13	0.18
	(1.09)	(0.77)	(0.56)	(0.45)	(0.35)	(0.25)	(0.21)	(0.22)	(0.21)	(0.19)
B: Controlling for relative interest-rate differentials										
$\hat{S}^R$	-1.84	-1.53	-1.34	-1.31*	-1.39**	-1.19**	-0.89*	-0.55	-0.32	-0.25
	(1.55)	(1.05)	(0.83)	(0.76)	(0.65)	(0.58)	(0.47)	(0.39)	(0.37)	(0.36)
$\hat{\epsilon}$	0.97	0.68	0.28	0.10	-0.13	-0.09	0.17	0.31	0.29	0.21
	(1.40)	(0.97)	(0.73)	(0.60)	(0.51)	(0.41)	(0.31)	(0.29)	(0.26)	(0.25)
r_h^R	3.37*	1.63***	0.97***	0.58**	0.32*	0.21	0.18	0.13	0.07	0.01
	(1.84)	(0.62)	(0.35)	(0.25)	(0.19)	(0.15)	(0.11)	(0.09)	(0.08)	(0.07)

Notes: Coefficient estimates on the fitted relative yield curve slope $\hat{S}_t^R$ and residual $\hat{\epsilon}$, estimated from regression (17), with the (log) ERRP as dependent variable. Regressions estimated using pooled end-of-month data for 6 currencies (AUD, CAD, CHF, EUR, JPY, GBP) against the USD. Log returns are annualised. All regressions include country fixed effects. The panel is unbalanced and Driscoll and Kraay (1998) standard errors are reported in parentheses. $*$, $**$ and $***$ denote significant point estimates at 10%, 5% and 1% levels, respectively.

Figure 5: Estimated coefficients for fitted relative slope and residual from ERRP regressions across holding periods when controlling for relative spot-yield differentials



Notes: $\hat{\gamma}_{2,h}$ (maroon circles) and $\hat{\gamma}_{3,h}$ (grey crosses) estimates from regression (18) for exchange-rate risk premia. Horizontal axis denotes the holding period h in months. Regressions estimated using pooled monthly data for 6 currencies (AUD, CAD, CHF, EUR, JPY, GBP) against the USD from 1980:01 to 2019:12, including country fixed effects. 95% confidence intervals, calculated using Driscoll and Kraay (1998) standard errors, denoted by shaded areas/bars around point estimates.

ods in this specification, the fitted relative slope coefficient is significant at 2 to 3-year horizons. This suggests that movements in the relative yield-curve slope attributable to changes in relative macroeconomic expectations explain variation in ERRP at these intermediate horizons, over and above spot-yield differentials.

5 Implications for International Asset Pricing and 'Disconnect'

Having shown that asymmetries in business-cycle expectations partly underlie the association between relative yield-curve slopes and ERRP, we build a minimal model of the term structure of interest rates and exchange rates which jointly rationalizes the predictability of exchange rates by the relative yield-curve slope, and the disconnect with interest rates.

5.1 Implied SDF Restrictions From Preference-Free Setup

A key building block for our model is the following restriction on the moments of SDFs implied by the regression evidence from Sections 2 and 3. We define innovations $\tilde{\epsilon}$ such that:

$$\tilde{\epsilon} = \left\{ \epsilon : \text{proj}(S_t^* - S_t | \epsilon) > 0 \ \& \ \text{proj}(r_{t,\kappa}^* - r_{t,\kappa} | \epsilon) = 0 \text{ for some } \kappa \geq 1 \right\} \tag{19}$$

This captures the logic that, following an increase in $\tilde{\epsilon}$, the relative slope becomes steeper but interest-rate differentials of some maturity κ are unchanged. What restrictions on the dynamics of SDFs deliver this within the preference-free setting outlined in Section 4? The Euler (10) implies that movements in the conditional mean and variance of SDFs must perfectly offset for these innovations to not drive interest-rate differentials:

$$\text{proj}(\mathbb{E}_t[m_{t,t+\kappa}] \ |\tilde{\epsilon}_t) = -\text{proj}(\mathcal{L}_t(M_{t,t+\kappa}) \ |\tilde{\epsilon}_t) \tag{20}$$

At the same time, by equation (16), these innovations must still drive the autocovariance of the SDF. This restriction is further supported by the fact that the yield curve predicts movements in exchange rates specifically through movements in the ERRP (i.e., $\text{proj}(\mathbb{E}_t[\Delta e_{t.t+\kappa}]|\tilde{\epsilon}) \approx -\text{proj}(\mathbb{E}_t[rx_{t+\kappa}^{FX}]|\tilde{\epsilon})$).[11] While a suite of general-equilibrium models can generate a negative relationship between the mean and variance of SDFs—notably, those with external habits (Campbell and Cochrane, 1999) and long-run risk (Bansal and Shaliastovich, 2013)—we go onto explain in Section 5.4 these models do not generally generate the association between the relative yield-curve slopes and ERRP.

5.2 A Stylized Model of Interest Rates and Exchange Rates

To consider this further, we use a minimal model of the term structure of interest rates and exchange rates. Our model builds on Backus, Chernov, and Zin (2014) who consider a single transitory risk factor, denoted by $\mathbb{T}$, which is fully spanned by interest rates. Relative to this, we extend the model with two (partly) unspanned factors. The first is a permanent factor, denoted by $\mathbb{P}$, which drives exchange-rate volatility, but is not reflected in the term structure of interest rates, consistent with Alvarez and Jermann (2005) and Chernov and Creal (2023) show. The second is an unspanned transitory factor, denoted by d, which we show is necessary

[11] Hassan, Mertens, and Wang (2024) emphasize the tension between models that generate a negative functional relationship between the mean and variance of SDFs and the unpredictability of exchange rates. Given that we are specifically focusing on the predictable component, orthogonal to short rates, our results are consistent.

for explaining the relationship between ERRP and the relative slope and, in contrast to $\mathbb{P}$, will appear in the the slope of the yield curve.

Let the (log) one-period SDF of the representative Home investor be given by:

$$m_{t,t+1} = m^{\mathbb{T}}_{t,t+1} + m^{\mathbb{P}}_{t,t+1} \tag{21}$$

We assume the (truncated) Wold decompositions of the transitory and permanent components can, respectively, be written as:

$$m^{\mathbb{T}}_{t,t+1} = \log\beta - \frac{1}{2}\gamma\sigma^2_{\mathbb{T}} + \alpha_0\epsilon_{\mathbb{T},t+1} + \alpha_1\epsilon_{\mathbb{T},t} \underbrace{-\frac{1}{2}\delta\sigma^2_d + d_0\epsilon_{d,t+1} + d_1\epsilon_{d,t}}_{\text{unspanned transitory factor}} \tag{22}$$

$$m^{\mathbb{P}}_{t,t+1} = -\frac{1}{2}\sigma^2_{\mathbb{P}} + \epsilon_{\mathbb{P},t+1} \tag{23}$$

where $\epsilon_{i,t}$ denote shocks to risk factors $i = \mathbb{T}, d, \mathbb{P}$ which we assume are uncorrelated and σ^2_i represent the corresponding constant volatilities. The transitory component is purged of permanent risk when $\alpha_0 = -\alpha_1$ and $d_0 = -d_1$.[12] Foreign variables are defined analogously and denoted with asterisks. We consider a model with symmetric factor loadings $\alpha_i = \alpha^*_i$, but allow for asymmetry in factor volatilities (i.e., $\sigma_{\mathbb{T}} \neq \sigma^*_i$ for $i = \mathbb{T}, d, \mathbb{P}$). For simplicity, we begin by assuming internationally complete markets, i.e., equation (12) with $\eta_{t,t+\kappa} = 0$, but Appendix D.1.1 shows this does not drive our results. We then analyze comparative statics with respect to the factor volatilities σ^2_i for $i = \mathbb{T}, d, \mathbb{P}$.

Combining (10) with (21), the κ-period bond yield can be written as:

$$\begin{aligned} -y_{t,\kappa} = & \log\beta + \frac{1}{\kappa}\left[(d_0^2 - \kappa\delta) + (d_1 + d_0)^2(\kappa - 1)\right]\frac{\sigma^2_d}{2} \\ & + \frac{1}{\kappa}\left[(\alpha_0^2 - \kappa\gamma) + (\alpha_1 + \alpha_0)^2(\kappa - 1)\right]\frac{\sigma^2_{\mathbb{T}}}{2} \end{aligned} \tag{24}$$

The short-term interest rate $r_t = y_{t,1}$ is inversely related to volatility as long as $\alpha_0^2 - \gamma > 0$ (i.e., precautionary savings motives dominate), consistent with models of habits and long-run risk (e.g., Engel, 2016). This generates the correct sign for UIP deviations, since excess returns to Foreign currency are positive when Home volatility is high.

Critically, choosing $\delta = d_0^2$ will impose (20) for $n = 1$, such that σ_d is not reflected in the short-term rate. However, it is still captured in the yield-curve slope, $S^{(\kappa)}_t = y^{(\kappa)}_t - r_t$:

$$S_t = \left(1 - \frac{1}{\kappa}\right)\left[d_0^2 - (d_1 + d_0)^2\right]\frac{\sigma^2_d}{2} + \left(1 - \frac{1}{\kappa}\right)\left[\alpha_0^2 - (\alpha_1 + \alpha_0)^2\right]\frac{\sigma^2_{\mathbb{T}}}{2} \tag{25}$$

This reflects asymmetries in both transitory innovations, but not in permanent innovations, and so leaving scope for the relative slope to have explanatory power over and above the short rate.

Evaluating (13), the ERRP reflects asymmetries in both transitory and permanent risk

[12] Alvarez and Jermann (2005) note that in the absence of permanent risk, the bond premium coincides with half the variance of the SDF, yielding the restrictions above.

across countries, as well as the hidden factor:

$$\begin{aligned}\mathbb{E}_t[rx^{FX}_{t+\kappa}] &= \frac{1}{\kappa}\left[d_0^2+(d_1+d_0)^2(\kappa-1)\right]\left(\frac{\sigma_d^2}{2}-\frac{\sigma_d^{*\,2}}{2}\right)\\ &+\frac{1}{\kappa}\left[\alpha_0^2+(\alpha_1+\alpha_0)^2(\kappa-1)\right]\left(\frac{\sigma_{\mathbb{T}}^2}{2}-\frac{\sigma_{\mathbb{T}}^{*\,2}}{2}\right)+\left(\frac{\sigma_{\mathbb{P}}^2}{2}-\frac{\sigma_{\mathbb{P}}^{*\,2}}{2}\right) \qquad (26)\end{aligned}$$

The following proposition summarizes our key result:

Proposition 1 (Risk, Bond Yields and ERRP) *Only an increase in the volatility of the hidden transitory factors (σ_d) is associated with a relatively steeper yield-curve slope and a higher ERRP, without being reflected in interest-rate differentials.*

Proof: Follows directly by using (24), imposing $\delta = d_0^2$, and then comparing with (25) and (26). □

More generally, the model implies that as the prominence of $\mathbb{P}$-risk rises (i.e., $\frac{\sigma_{\mathbb{P}}}{\sigma_{\mathbb{T}}+\sigma_d+\sigma_{\mathbb{P}}}\uparrow$), the relationship between either yields or yield curves and the ERRP becomes less pronounced, with the R^2 of a regressions (1) or (2) approaching 0 in the limit. In contrast, the opposite is true as $\mathbb{T}$-risk becomes more important ($\frac{\sigma_{\mathbb{P}}}{\sigma_{\mathbb{T}}+\sigma_d+\sigma_{\mathbb{P}}}\uparrow$). However, only as d-risk rises ($\frac{\sigma_d}{\sigma_{\mathbb{T}}+\sigma_d+\sigma_{\mathbb{P}}}\uparrow$) does the yield curve become an important explanatory variable for ERRP that is orthogonal to yield differentials.

Below, we consider an illustrative three-horizon example (short-, medium- and long-run) where the predictability of the cross-country yield-curve slope is highest in the medium horizon, recalling our empirical findings. To this end, we generalize the framework such that the choice of δ imposes (20) for $\kappa > 1$, which ensures risk σ_d is unspanned in longer-maturity interest rates. Then, short-term rates can have some predictive power over exchange rates in line with the UIP puzzle.[13] The following proposition summarizes our key finding:

Proposition 2 (Horizon Variation) *Assume: (i) permanent risk is equalized across countries ($\sigma_{\mathbb{P}} = \sigma_{\mathbb{P}}^*$, $\alpha_0 \approx -\alpha_1, d_0 \approx -d_1$), and (ii) $\delta = \frac{1}{\kappa}\left(d_0^2+(d_0+d_1)^2(\kappa-1)\right)$. While a steeper relative yield-curve slope is associated with higher currency returns at both horizons 1 and $\overline{\kappa} < \infty$, the relative slope reflects proportional information to the spot-yield differential at $t+1$, but orthogonal information at $t+\overline{\kappa}$. In the long run, ERRP are zero.*

Proof: $\delta = \frac{1}{\kappa}\left(d_0^2+(d_0+d_1)^2(\kappa-1)\right)$ imposes (20) for $\kappa = \overline{\kappa}$—i.e., at intermediate horizons. The equalization of the permanent risk factor ($\sigma_{\mathbb{P}} = \sigma_{\mathbb{P}}^*$), along with the purging of permanent risk from the transitory factors ($\alpha_0 \approx -\alpha_1, d_0 \approx -d_1$) ensures ERRP are zero in the long-run and so there is no predictability. □

[13] As shown by Lustig et al. (2019), carry-trade returns at long horizons are zero (and thus unpredictable) if there are no differences in the permanent innovations of SDFs across countries. Notably, Andrews, Colacito, Croce, and Gavazzoni (2024) find evidence that $rx^{CT\,(\infty)} = 0$ only because it is negative before and positive post-GFC. Consistently, Chernov and Creal (2023) argue the evidence for zero ERRP at long horizons appears weak, and the power of the test is low.

5.3 Endogenizing Hidden Factors via Domestically Incomplete Markets

Term-structure models like above are almost exclusively written under complete markets where there can only exist a unique SDF. We show hidden factors, as above, arise as equilibrium outcomes in a large class of incomplete-markets models. The generalized model is consistent with no-arbitrage pricing by the SDF m_{t+1} (21), but the structure of risk is constrained due to trade in assets by an additional investor with SDF $\tilde{m}_{t+1}$—ruled out if domestic financial markets were complete. Our framework relates to models of incomplete markets and limited participation such as Guvenen (2009) and Marin and Singh (2024), as well as models of preferred habitat as in Gourinchas et al. (2022) and Greenwood et al. (2023).

Suppose the second investor, with SDF $\tilde{m}_{t,t+1}$, is active in domestic markets, but only trades in domestic bonds with maturities $\kappa \in \tilde{\boldsymbol{\kappa}}$. Their SDF is defined by:[14]

$$\tilde{m}_{t+1} = \quad \log\beta - \frac{1}{2}\sigma_{\mathbb{T}}^2 + \alpha_0\epsilon_{\mathbb{T},t+1} + \alpha_1\epsilon_{\mathbb{T},t}$$

Critically, both investors m_{t+1} and $\tilde{m}_{t+1}$ trade in the risk-free bonds of maturities in $\tilde{\boldsymbol{\kappa}}$ which imposes tight restrictions on the structure of risk, summarized in the following proposition:

Proposition 3 (Incomplete Markets and Hidden Factors) *Suppose $\tilde{\boldsymbol{\kappa}} = 1$ such that only the risk-free rate is traded by both m and $\tilde{m}$. No arbitrage requires $\delta = d_0^2$. More generally, for any $\kappa \in \tilde{\boldsymbol{\kappa}}$ maturity traded, condition (ii) from Proposition 2 is required to satisfy no-arbitrage.*

Proof: Consider the set of conditions $\mathbb{E}[M_{t,t+\kappa}] = \mathbb{E}[\tilde{M}_{t,t+\kappa}] = 1/R_t^{(\kappa)}\ \forall\ \kappa \in \tilde{\boldsymbol{\kappa}}$ which reflect risk-sharing between agents at horizon $\tilde{\kappa}$. Once again, under complete markets $\tilde{\boldsymbol{\kappa}} = \mathbb{R}$, which implies the mean (and variance) of all multi-horizon SDFs is equalized. $\square$

Intuitively, bonds traded by both investors only price risks which both investors face and agree upon. In the spirit of preferred-habitat models, $\epsilon_{d,t+1}$ can reflect investor-specific demand shocks, which drive both the term structure of interest rates and exchange rates. Since $\tilde{m}_{t+1}$ does not face these habitat shocks, the only equilibrium consistent with no arbitrage requires δ such that (20) holds.

Heterogeneous Expectations. Alternatively, domestic market incompleteness may arise because investors have heterogeneous expectations over the same fundamentals. To consider this, define a new subjective expectation operator $\tilde{\mathbb{E}}_t$, distinct from the objective expectations above, such that:

$$\mathbb{E}_t[\epsilon_{d,t+1}^2] = \sigma_d^2, \quad \tilde{\mathbb{E}}_t[\epsilon_{d,t+1}^2] = (1-\omega)\sigma_d^2, \quad \omega < 1 \tag{27}$$

and, for simplicity, assume $\tilde{m}_{t+1}$ is given by (22) with $-\frac{1}{2}\delta\sigma_d^2$ replaced by $-\frac{1}{2}\delta\sigma_d^2(1-\omega)$. Investors agree on risks in the limit $\tilde{\omega} \to 0$, and markets are de-jure complete. In contrast, as $\omega \to 1$, the second investor underestimates the d-factor and their SDF converges to (27). In this case, the

[14] Notice this is a normalization. More generally, we can consider $m_{t,t+1}^i = log(D_{t,t+1}^i M_{t,t+1})$ and interpret the expectations above as cross-sectional (see also Constantinides and Duffie, 1996; Marin and Singh, 2024).

only admissible equilibrium with trade in the one-period bond ($\tilde{\boldsymbol{\kappa}} = 1$) has a hidden factor, per Proposition 3.

5.4 A Dynamic Asset-Pricing Model

Finally, we show our results generalize to a canonical dynamic asset-pricing model with stochastic volatility, building on a two-country version of the Cox et al. (1985) model, studied in Backus et al. (2001) and Lustig et al. (2019), amongst others. We relegate details and derivations to the Appendix D.2.

Consider a representative Home investor's SDF which loads on three independent country-specific factors $z_{0,t}, z_{1,t}, z_{2,t}$:

$$\begin{aligned} -m_{t,t+1} &= z_{0,t} + (\lambda_1^2/2 - 1)z_{1,t} + \lambda_1\sqrt{z_{1,t}}\epsilon_{t+1} + (\lambda_2^2/2)z_{2,t} + \lambda_2\sqrt{z_{2,t}}\epsilon_{2,t+1}, \\ z_{i,t+1} &= (1-\phi_i)z_{2,t} + \phi_i z_{i,t} - \sigma_i\sqrt{z_{i,t}}\epsilon_{i,t+1}, \quad \text{for } i \in \{0,1\} \\ z_{2,t+1} &= (1-\phi_2)\theta \ + \phi_2 z_{2,t} - \sigma_2\sqrt{z_{2,t}}\epsilon_{2,t+1} \end{aligned} \tag{28}$$

where $\lambda_1, \lambda_2 < 0$ are coefficients which capture the price of risk with respect to each factor.[15,16] For simplicity, $z_{0,t}$ has a zero price of risk. We assume that the representative Foreign investor's SDF $m^*_{t,t+1}$ and country-specific pricing factors $z^*_{i,t}$ are defined analogously, and with symmetric coefficients ($\lambda^*_i = \lambda_i$, $\phi^*_i = \phi_i$, and $\sigma^*_i = \sigma_i$).

Assuming log-normality, combining (28) and the expression for the (log) price of an n-period bond, $p_{t,n} = \mathbb{E}_t[m_{t,t+1} + p_{t+1,n-1}] + (1/2)\text{var}_t(m_{t,t+1} + p_{t+1,n-1})$, we can write the the (log) bond price as an affine function of pricing factors $p_{t,n} = -\left(\Omega_n + A_n z_{0,t} + B_n z_{1,t} + C_n z_{2,t}\right)$, where Ω_n, A_n, B_n and C_n are recursively defined.

Term Structure. An immediate consequence is that short rates are given by $r_t^{(*)} = z_{0,t}^{(*)} - z_{1,t}^{(*)}$ and are countercyclical with respect to z_1 implying $B_n < 0$, consistent with Verdelhan (2010). Absent additional factors, this counterfactually implies a negative average slope of the yield curve and bond premium (Wachter, 2006) and increasingly negative longer-horizon UIP deviations (Lustig et al., 2019). The *ex ante* bond risk premium is given by:

$$\begin{aligned} \mathbb{E}_t\left[rx_{t,t+1}^{(\infty)}\right] + \frac{1}{2}\text{var}_t(r_{n,t+1}) &= -\,\text{cov}_t(p_{t+1,n-1}, m_{t,t+1}) \\ &= \underbrace{-\lambda_1\sigma_1 B_{n-1}}_{<0} z_{1,t} - \lambda_2\sigma_2 C_{n-1} z_{2,t} \end{aligned} \tag{29}$$

and additionally depends on $\epsilon_{2,t}$ which is hidden from r_t.

[15] A negative price of risk is required in the one-factor model (Backus et al., 1998) and two-country multi-factor models (Backus et al., 2001; Lustig and Verdelhan, 2019).

[16] This setup resembles the the model of central tendency in Balduzzi et al. (1998) where $z_{2,t}$ captures the long-run mean of the short rate. Ang and Chen (2010) provide direct evidence for the importance of a time-varying long run mean. Kozicki and Tinsley (2001) identify monetary policy and long run expectations as a good proxy for $z_{2,t}$, providing a supporting economic interpretation of this factor.

Hidden Factors in CIR. Once again, the SDF (28) can be understood as an equilibrium outcome from an incomplete markets model. Define a SDF $\tilde{m}_{t,t+1}$ which is identical to $m_{t,t+1}$ except that it does not depend on $z_{2,t}$, i.e. $\tilde{\lambda}_2 = 0$. Then if both investors trade the short bond, no-arbitrage requires $C_0 = 0$, which is satisfied in the model above. More generally, one can express the model with a conditional mean of $(\lambda_1^2/2 - 1)z_{1,t} + (\lambda_2^2/2 + \zeta)z_{2,t}$. Then, ζ must be 0 to satisfy no arbitrage. Critically, however, there are additional requirements for subsequent C_n to be non-zero, specific to the dynamic model, and detailed below.

Proposition 4 (Bond Yields and ERRP in CIR) *In the model described by (28), a relatively steep Home yield-curve slope ($S_t > S_t^*$) can be associated with higher future ERRP ($rx_{t,t+\kappa}^{FX}$), orthogonal to short-rate differentials, only if the loading on $z_{2,t}$ is positive at longer maturities ($C_n > 0$) which, in turn, requires $z_{0,t}$ to tend to $z_{2,t}$.*

Proof: Assuming $\lambda_1, \lambda_2 < 0$, and given that $B_{n-1} < 0$, (29) delivers a positive bond premium only if $C_{n-1} > 0$ for $n > 1$. Given $C_0 = 0$ so that $z_{2,t}$ is unspanned in risk-free rates, inspecting the recursion for C_n, $C_{n-1} > 0$ is only possible because $\mathbb{E}[z_{0,t+1}] = z_{2,t}$. □

The sufficient condition to deliver a positive yield-curve slope requires $z_{2,t}$ to be persistent ($\phi_2 > \phi_1$), which can also rationalize why the relative yield-curve slope especially matters at intermediate (as opposed to short or long) horizons.[17] In the short run, if $|\lambda_2| < |\lambda_1|$, the short-rate differential can be a good predictor of ERRP since the hidden factor is less important. However, due to differences in the persistence of factors and the central tendency, the relative slope becomes an orthogonal predictor of future and longer horizon ERRP. ERRP and its predictability will approach zero asymptotically if permanent innovations are ruled out.

6 Conclusion

Overall, our paper highlights that a significant component of currency fluctuations and ERRP, at business-cycle horizons in particular, can be explained by cross-country differences in the term structure of interest rates. Preference-free results derived assuming no-arbitrage suggest this is driven by cross-country differences in the autocorrelation of investor valuations (SDFs) across countries, consistent with a notion of transitory business-cycle risk. Driven by this insight, we find evidence that survey data on expectations for GDP and inflation explain relative yield-curve slopes. In turn, regressing exchange-rate movements on the fitted component of the relative yield-curve slope, we find that cross-country asymmetries in macroeconomic expectations are a significant determinant of ERRP, orthogonal to interest rates, especially at 2 to 4-year horizons.

In addition to finding evidence that currency fluctuations reflect expectations of macroeconomic fundamentals, we illustrate the importance of transitory factors which can be 'hidden' from short-term interest rates because of offsetting effects on the conditional mean and variance of SDFs. As such, these factors are consistent with the literature on the exchange-rate 'disconnect', while also being captured by the yield-curve slope. Going a step further, we propose

[17] We generalize the SDF process to allow factors to be hidden from longer maturities and provide details in Apppendix D.2.1.

market incompleteness as a plausible mechanism by which hidden factors arise as endogenous outcomes in equilibria with trade.

While our results further the understanding of exchange rates within standard modelling frameworks, a limitation of our analysis is that there appears to be little scope for forecasting. This is predominantly because the relationship we document between the relative yield-curve slope and ERRP spans longer horizons, compounding forecast errors. Additionally, while our theoretical framework speaks to *ex ante* ERRP, our regressions use realized currency moves because of the lack of longer-horizon market-based forecasts.

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Appendix

A Data Sources

We use nominal zero-coupon government bond yields at maturities from 6 months to 10 years for 7 industrialised countries: US, Australia, Canada, Euro Area, Japan, Switzerland and UK. Our benchmark sample spans 1980:01-2019:12, although the panel of interest rates is unbalanced as bond yields are not available from the start of the sample in all jurisdictions. Table A1 summarises the sources of nominal zero-coupon government bond yields, and the sample availability, for the benchmark economies in our study.

Table A1: Yield Curve Data Sources

Country	Sources	Start Date
US	Gürkaynak, Sack, and Wright (2007)	1971:11
Australia	Reserve Bank of Australia	1992:07
Canada	Bank of Canada	1986:01
Euro Area	Bundesbank (German Yields)	1980:01
Japan	Wright (2011) and Bank of England	1986:01
Switzerland	Swiss National Bank	1988:01
UK	Anderson and Sleath (2001)	1975:01

Notes: Data from before 1980:01 are not used in this paper.

Exchange rate data is from *Datastream*, reflecting end-of-month spot rates *vis-à-vis* the US dollar. Liquidity yields are from Du et al. (2018), available at the 1, 2, 3, 5, 7 and 10-year maturities. The earliest liquidity yields are available from 1991:04 for some countries (e.g. UK). The latest liquidity yields are available from 1999:01 (e.g. euro area). For both exchange rates and liquidity yields, we use end-of-month observations.

B Robustness: Exchange Rates and the Yield Curve Slope

B.1 Full Results from UIP and Yield Curve-Augmented Regressions

Table B1 tabulates our benchmark results for regressions (1) and (2), which are discussed in Section 2. Columns (1)-(2) present the $\beta_{1,\kappa}$ estimates and within R^2 statistics, at different horizons, from the canonical UIP regression. Columns (3)-(5) present $\beta_{1,\kappa}$ and $\beta_{2,\kappa}$ estimates, along with the within R^2 from the slope-augmented regression.

B.2 Robustness Results for Yield Curve-Augmented Regressions

We discuss each robustness exercise in turn.

Relative Curvature. Adding a proxy for the relative curvature to regression (2) does not significantly alter conclusions around the relative yield-curve slope, as columns (1)-(4) of Table B2 show. There remains a tent-shaped relationship on relative slope coefficients across horizons. The relative curvature coefficient has a negative tent-shaped relationship across horizons, but this finding is not robustly significant.[18]

Predictability of Interest Rates. The inclusion of interest rates in specification (2) poses a potential challenge, as interest rates are persistent and have a factor structure that is a function of the yield-curve slope. To ensure that the relationship between the slope and ERRP is not driven by the predictability of, and correlation with, interest rates, we also estimate a simple regression of exchange-rate changes on the relative slope, omitting return differentials. These results, shown in columns (5)-(6) of Table B2, as well as a specification where we include the relative yield curve level alongside slope (and curvature) as in Chen and Tsang (2013), indicate that the tent-shaped relationship across horizons is robust to these changes.

Sub-Sample Stability. Our main results are robust to splitting the sample into two sub-periods, as columns (7)-(10) of Table B2 show. First, a pre-global financial crisis sample (1980:01-2008:06), which excludes the period in which central banks engaged in unconventional monetary policies. Second, a sample covering the post-crisis period (1990:01-2019:12), in which there was a crash in carry trade around 2008 and a switch in UIP coefficients (Bussière, Chinn, Ferrara, and Heipertz, 2022) where there still exists a significant relationship with the relative yield-curve slope, but it is shifted to later horizons.

Long-Horizon Inference. In long-horizon variants of regressions (1) and (2), the number of non-overlapping observations can be limited. Therefore, size distortions—i.e. the null hypothesis being rejected too often—are a pertinent concern, especially with small samples and persistent regressors (Valkanov, 2003). To carry out more conservative inference, we draw on

[18] Using more conservative standard errors, described in the subsequent 'Long-Horizon Inference' paragraph, we do not find a significant relationship between exchange rate changes and the relative curvature across horizons.

Table B1: Coefficient estimates from canonical UIP regression and regression augmented with relative yield curve slope

Maturity	(1)	(2)	(3)	(4)	(5)
	UIP Regression		Slope-Augmented Regression		
κ	$r^*_\kappa - r_\kappa$	Within R^2	$r^*_\kappa - r_\kappa$	$S^* - S$	Within R^2
6 months	-1.31*	0.0191	-0.89	0.29	0.0197
	(0.74)		(1.14)	(0.71)	
12 months	-1.29**	0.0323	-0.83	0.62	0.0341
	(0.57)		(0.90)	(1.07)	
18 months	-1.21**	0.0385	-0.39	1.65	0.0494
	(0.49)		(0.72)	(1.19)	
24 months	-0.99**	0.0308	-0.14	2.21*	0.049
	(0.42)		(0.65)	(1.32)	
30 months	-0.77**	0.0222	0.13	2.92**	0.0528
	(0.35)		(0.59)	(1.38)	
36 months	-0.54*	0.013	0.27	3.16**	0.0492
	(0.31)		(0.51)	(1.30)	
42 months	-0.25	0.0033	0.38	2.92***	0.0354
	(0.30)		(0.41)	(1.08)	
48 months	0.03	0.0001	0.49	2.45**	0.023
	(0.28)		(0.33)	(0.96)	
54 months	0.36	0.0085	0.73***	2.25**	0.0285
	(0.24)		(0.26)	(0.95)	
60 months	0.61***	0.0261	0.94***	2.31**	0.0468
	(0.23)		(0.24)	(1.05)	
66 months	0.83***	0.052	1.14***	2.46**	0.0755
	(0.21)		(0.23)	(1.11)	
72 months	1.00***	0.0815	1.25***	2.35**	0.103
	(0.18)		(0.19)	(1.04)	
78 months	1.04***	0.096	1.22***	1.96**	0.111
	(0.15)		(0.17)	(0.91)	
84 months	1.00***	0.0965	1.12***	1.56*	0.106
	(0.15)		(0.16)	(0.86)	
90 months	0.91***	0.0866	0.99***	1.12	0.0916
	(0.15)		(0.16)	(0.87)	
96 months	0.77***	0.067	0.81***	0.55	0.0682
	(0.15)		(0.16)	(0.86)	
102 months	0.61***	0.0436	0.61***	-0.02	0.0436
	(0.15)		(0.16)	(0.94)	
108 months	0.47***	0.027	0.44***	-0.61	0.0284
	(0.15)		(0.16)	(1.02)	
114 months	0.42***	0.022	0.38**	-0.86	0.0246
	(0.15)		(0.16)	(1.07)	
120 months	0.42***	0.0233	0.37**	-1.23	0.0286
	(0.15)		(0.16)	(1.11)	

Notes: Columns (1)-(2) present results from canonical UIP regression (1), a regression of κ-period exchange-rate change $\Delta^\kappa e_{t+\kappa}$ on the κ-period return differential $r^*_{t,\kappa} - r_{t,\kappa}$. Columns (3)-(5) present results from extended regression (2), using relative yield-curve slope $S^*_t - S_t$ as an additional regressor. Regressions estimated using pooled end-of-month data for 6 currencies (AUD, CAD, CHF, EUR, JPY, GBP) against the USD from 1980:01 to 2019:12, including country fixed effects. The panel is unbalanced. $*$, $**$ and $***$ denote statistical significance at the 10%, 5% and 1% levels, respectively, using Driscoll and Kraay (1998) standard errors (reported in parentheses).

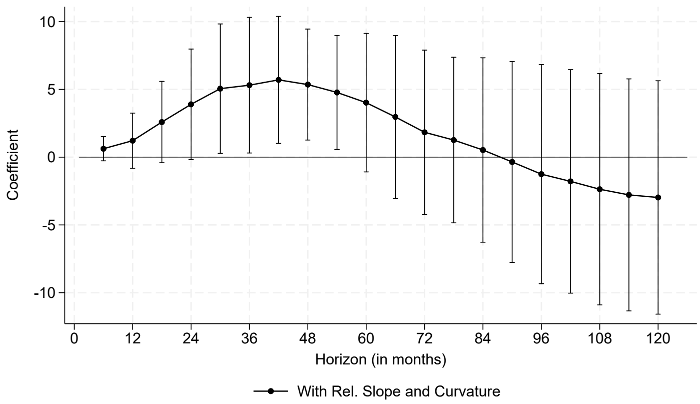
Table B2: Robustness of coefficient estimates from augmented UIP regressions

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
	Slope & Curvature-Aug. Reg.				Slope Only		Pre-GFC		Post-GFC	
	$r^*_\kappa - r_\kappa$	$S^* - S$	$C^* - C$	Within R^2	$S^* - S$	Within R^2	$r^*_\kappa - r_\kappa$	$S^* - S$	$r^*_\kappa - r_\kappa$	$S^* - S$
6 months	-0.85	0.62	-0.62	0.0212	0.77*	0.0177	-1.01	0.54	0.12	0.35
	(1.14)	(0.77)	(0.77)		(0.45)		(1.56)	(0.92)	(1.22)	(0.62)
12 months	-0.70	1.22	-0.95	0.0358	1.41**	0.0300	-0.68	0.93	-0.84	-0.18
	(0.88)	(1.24)	(1.08)		(0.66)		(1.21)	(1.41)	(0.81)	(0.78)
18 months	-0.21	2.59*	-1.46	0.0521	2.13***	0.0479	-0.02	2.04	-0.77	0.21
	(0.73)	(1.49)	(1.23)		(0.79)		(0.96)	(1.58)	(0.68)	(0.95)
24 months	0.12	3.90**	-2.62	0.0556	2.42***	0.0487	0.26	2.74	-0.54	0.61
	(0.66)	(1.75)	(1.66)		(0.87)		(0.84)	(1.72)	(0.62)	(1.13)
30 months	0.42	5.05***	-3.36*	0.0621	2.71***	0.0525	0.51	3.55**	-0.16	1.40
	(0.60)	(1.83)	(1.94)		(0.85)		(0.77)	(1.77)	(0.53)	(1.22)
36 months	0.52	5.31***	-3.46*	0.0583	2.69***	0.0475	0.51	3.61**	-0.07	1.58
	(0.55)	(1.76)	(1.80)		(0.79)		(0.68)	(1.70)	(0.43)	(1.16)
42 months	0.67	5.70***	-4.58***	0.0505	2.22***	0.0307	0.51	3.39**	0.01	1.34
	(0.45)	(1.52)	(1.76)		(0.77)		(0.57)	(1.45)	(0.32)	(0.89)
48 months	0.75**	5.36***	-4.91***	0.0396	1.53*	0.0138	0.43	2.89**	0.30	1.29
	(0.35)	(1.24)	(1.63)		(0.85)		(0.46)	(1.24)	(0.26)	(0.85)
54 months	0.92***	4.77***	-4.37***	0.0414	0.89	0.00447	0.58	2.63**	0.75***	1.60
	(0.27)	(1.21)	(1.61)		(0.96)		(0.36)	(1.07)	(0.24)	(0.97)
60 months	1.05***	4.02***	-3.04*	0.0528	0.56	0.00166	0.77**	2.76**	1.15***	1.88*
	(0.24)	(1.39)	(1.70)		(1.12)		(0.31)	(1.14)	(0.25)	(1.12)
66 months	1.17***	2.97*	-0.91	0.0760	0.38	0.000714	0.97***	2.90**	1.43***	2.16*
	(0.23)	(1.56)	(1.90)		(1.23)		(0.28)	(1.20)	(0.24)	(1.19)
72 months	1.23***	1.84	0.96	0.104	0.11	6.00e-05	1.08***	2.54**	1.57***	2.29*
	(0.19)	(1.50)	(1.83)		(1.23)		(0.24)	(1.14)	(0.23)	(1.17)
78 months	1.19***	1.26	1.31	0.112	-0.16	0.000118	1.05***	2.05*	1.47***	1.88*
	(0.17)	(1.46)	(1.90)		(1.11)		(0.21)	(1.06)	(0.21)	(0.99)
84 months	1.09***	0.53	1.97	0.109	-0.33	0.000500	0.94***	1.51	1.37***	1.38
	(0.15)	(1.56)	(2.22)		(1.00)		(0.21)	(1.05)	(0.18)	(0.87)
90 months	0.95***	-0.35	2.84	0.0963	-0.49	0.00108	0.75***	0.85	1.26***	0.87
	(0.15)	(1.65)	(2.32)		(0.99)		(0.21)	(1.02)	(0.17)	(0.89)
96 months	0.77***	-1.25	3.51	0.0753	-0.71	0.00222	0.52***	-0.07	1.04***	0.15
	(0.15)	(1.74)	(2.50)		(0.94)		(0.20)	(0.98)	(0.18)	(0.87)
102 months	0.58***	-1.79	3.47	0.0504	-0.93	0.00366	0.28	-0.83	0.80***	-0.65
	(0.15)	(1.72)	(2.51)		(0.99)		(0.18)	(1.00)	(0.21)	(0.95)
108 months	0.42***	-2.37	3.48	0.0348	-1.25	0.00621	0.10	-1.47	0.59***	-1.41
	(0.15)	(1.73)	(2.58)		(1.04)		(0.18)	(1.05)	(0.22)	(1.01)
114 months	0.37**	-2.78	3.84	0.0320	-1.39	0.00723	0.08	-1.70	0.46**	-1.79*
	(0.16)	(1.69)	(2.66)		(1.07)		(0.18)	(1.10)	(0.22)	(1.02)
120 months	0.36**	-2.97*	3.48	0.0346	-1.73	0.0110	0.11	-1.78	0.39*	-2.22**
	(0.16)	(1.65)	(2.75)		(1.11)		(0.18)	(1.15)	(0.21)	(1.05)

Notes: Columns (1)-(4) present results from UIP regression augmented with relative yield-curve slope and curvature over 1980:01-2019:12 sample. Columns (5)-(6) present results from regression of κ-period exchange-rate change $\Delta^\kappa e_{t+\kappa}$ on the relative slope only, over 1980:01-2019:12 sample. Columns (7)-(8) present results from UIP regression augmented with relative yield-curve slope over 1980:01-2008:06 sample. Columns (9)-(10) present results from UIP regression augmented with relative yield-curve slope over 1990:01-2019:12 sample. Regressions estimated using pooled end-of-month data for 6 currencies (AUD, CAD, CHF, EUR, JPY, GBP) against the USD, including country fixed effects. $*$, $**$ and $***$ denote statistical significance at the 10%, 5% and 1% levels, respectively, using Driscoll and Kraay (1998) standard errors (reported in parentheses).

Moon, Rubia, and Valkanov (2004) who propose the scaling of t-statistics by $1/\sqrt{\kappa}$, showing that these scaled statistics are approximately standard normal when regressors are highly persistent.[19] As Figure B1 shows, our primary result remains significant when using these more conservative t-statistics.

Figure B1: Estimated relative-slope coefficients with adjusted standard errors



Notes: Black curcles denote $\hat{\beta}_{2,\kappa}$ estimates from regression (2). Horizontal axes denote the horizon κ in months. Regressions estimated using pooled monthly data for 6 currencies (AUD, CAD, CHF, EUR, JPY, GBP) against the USD from 1980:01 to 2019:12, including country fixed effects. 90% confidence intervals, calculated using Valkanov (2003) and Moon et al. (2004) long-horizon standard errors, denoted by black bars around point estimates.

B.3 Alternative Currency Bases

The tent-shaped pattern for the relative slope coefficient appears specific to the USD currency base. This is shown in Table B3, which plots the coefficients on the relative slope when regression (2) is estimated with each alternative currency base in turn (i.e., AUD, CAD, CHF, EUR, JPY, GBP). For almost all currencies, the estimated coefficients on the relative are broadly insignificant at business-cycle horizons, and there is very little sign of a tent-shaped relationship with positive coefficients across horizons—except for the CHF.

[19] Because this is an approximate result, these standard errors are not our preferred metric for inference. Indeed, the scaled t-statistics tend to under-reject the null when regressors are not near-unit root, implying that these confidence bands offer some of the most conservative inference for our regressions.

Table B3: Relative slope coefficient from augmented UIP regression using alternative base currencies

Base:	(1) AUD $S^* - S$	(2) CAD $S^* - S$	(3) CHF $S^* - S$	(4) EUR $S^* - S$	(5) GBP $S^* - S$	(6) JPY $S^* - S$
6 months	0.02	0.27	0.69*	-0.49	-0.64	0.77
	(0.52)	(0.38)	(0.40)	(0.77)	(0.64)	(0.48)
12 months	0.28	0.60	1.31**	-1.19	-0.90	1.33*
	(0.73)	(0.54)	(0.61)	(1.27)	(0.90)	(0.74)
18 months	0.40	0.60	1.86**	-1.27	-0.58	1.50*
	(0.77)	(0.65)	(0.74)	(1.43)	(0.95)	(0.91)
24 months	0.35	0.65	2.09***	-1.00	-0.47	1.64
	(0.77)	(0.70)	(0.76)	(1.27)	(1.01)	(1.01)
30 months	0.20	0.35	2.38***	-1.09	-0.36	1.71
	(0.82)	(0.76)	(0.74)	(1.14)	(1.05)	(1.10)
36 months	0.05	-0.23	2.32***	-1.86*	-0.63	1.25
	(0.89)	(0.90)	(0.72)	(1.12)	(1.14)	(1.20)
42 months	-0.28	-1.24	2.07***	-2.77***	-1.13	0.46
	(0.99)	(0.96)	(0.75)	(1.05)	(1.23)	(1.30)
48 months	-0.70	-2.03**	1.37*	-3.70***	-1.76	-0.81
	(1.01)	(0.98)	(0.82)	(1.01)	(1.24)	(1.40)
54 months	-0.52	-2.43***	0.58	-3.71***	-1.88*	-1.55
	(0.97)	(0.93)	(0.86)	(1.08)	(1.14)	(1.40)
60 months	0.01	-2.25**	-0.03	-2.76**	-1.78	-1.74
	(0.91)	(0.94)	(0.84)	(1.33)	(1.16)	(1.45)
66 months	0.59	-1.68*	-0.52	-1.14	-1.59	-1.21
	(0.90)	(0.98)	(0.80)	(1.70)	(1.15)	(1.50)
72 months	1.07	-1.44	-1.11	0.57	-1.31	-0.55
	(0.94)	(0.97)	(0.73)	(1.88)	(0.94)	(1.47)
78 months	1.30	-0.86	-1.57**	2.02	-1.25	0.54
	(1.06)	(0.94)	(0.72)	(1.77)	(0.86)	(1.36)
84 months	1.54	-0.34	-1.81**	2.91*	-0.97	1.97
	(1.09)	(0.97)	(0.75)	(1.64)	(0.81)	(1.21)
90 months	2.00*	-0.29	-2.00**	3.27**	-0.68	2.58**
	(1.09)	(0.95)	(0.82)	(1.53)	(0.77)	(1.13)
96 months	2.20*	-0.74	-2.37***	3.01**	-0.88	2.45**
	(1.14)	(0.92)	(0.85)	(1.48)	(0.81)	(1.10)
102 months	2.29*	-1.47	-2.70***	2.35	-1.16	2.35**
	(1.20)	(1.01)	(0.91)	(1.54)	(0.90)	(1.05)
108 months	2.24*	-2.16**	-3.25***	1.63	-1.65*	2.21**
	(1.21)	(1.10)	(0.98)	(1.72)	(0.89)	(1.10)
114 months	1.86	-2.48**	-3.58***	1.41	-2.11**	1.97*
	(1.19)	(1.19)	(1.01)	(1.87)	(0.84)	(1.17)
120 months	1.30	-2.90**	-3.91***	1.14	-2.74***	1.09
	(1.18)	(1.26)	(1.03)	(2.04)	(0.90)	(1.16)

Notes: Coefficients on relative yield curve slope from extended regression (2), using relative yield-curve slope $S_t^* - S_t$ as an additional regressor, for different currency bases. Regressions estimated using pooled end-of-month data for 7 currencies (AUD, CAD, CHF, EUR, JPY, GBP, USD) from 1980:01 to 2019:12, including country fixed effects. The panel is unbalanced. $*$, $**$ and $***$ denote statistical significance at the 10%, 5% and 1% levels, respectively, using Driscoll and Kraay (1998) standard errors (reported in parentheses).

C Robustness: Excess Returns and the Yield-Curve Slope

In Table C1, we present the mean return from a simple investment strategy that goes long the Foreign bond and short the US bond when the Foreign yield curve slope is lower than the US yield curve slope, and goes long the US bond and short the Foreign bond when the US yield curve slope is lower than the Foreign yield curve slope. Relative to Lustig et al. (2019), we present the mean dollar-bond return differences for a range of holding periods $h = 6, 12, ..., 60$ and maturities $\kappa = 6, 12, ..., 120$ (in months).

Table C1: Mean Excess Returns from Dynamic Long-Short Bond Portfolios

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
	Holding Periods									
	6m	12m	18m	24m	30m	36m	42m	48m	54m	60m
Dollar-Bond Return Difference: $rx^{(\kappa),\$}_{j,t,t+h} - rx^{(\kappa)}_{US,t,t+h}$										
12m	1.95									
18m	1.81	2.48								
24m	1.70	2.38	3.04							
30m	1.60	2.3	2.98	3.3						
36m	1.49	2.21	2.92	3.26	3.30					
42m	1.38	2.12	2.85	3.22	3.27	3.08				
48m	1.26	2.01	2.76	3.16	3.24	3.06	2.9			
54m	1.15	1.91	2.67	3.10	3.20	3.03	2.88	2.57		
60m	1.03	1.81	2.58	3.03	3.15	2.99	2.85	2.55	2.30	
66m	0.93	1.72	2.49	2.95	3.09	2.95	2.82	2.52	2.28	2.35
72m	0.83	1.63	2.40	2.88	3.03	2.89	2.77	2.49	2.25	2.32
78m	0.74	1.55	2.32	2.81	2.96	2.84	2.72	2.45	2.22	2.29
84m	0.67	1.48	2.24	2.74	2.90	2.78	2.67	2.41	2.18	2.26
90m	0.58	1.41	2.17	2.67	2.84	2.72	2.62	2.36	2.14	2.23
96m	0.51	1.35	2.09	2.60	2.78	2.65	2.56	2.31	2.10	2.19
102m	0.45	1.29	2.03	2.54	2.71	2.59	2.50	2.26	2.06	2.16
108m	0.39	1.23	1.96	2.48	2.65	2.53	2.44	2.21	2.02	2.12
114m	0.34	1.18	1.90	2.42	2.59	2.47	2.39	2.16	1.98	2.09
120m	0.29	1.12	1.84	2.36	2.53	2.41	2.33	2.11	1.94	2.05

Notes: Summary return statistics from investment strategies that go long in the Foreign-country bond and short in the US bond when the Foreign yield curve slope is lower than the US yield curve slope, and go long in the US bond and short in the Foreign-country bond when the Foreign yield curve slope is higher than the US yield curve slope. The table reports the mean US dollar-bond excess return difference for different holding periods and different maturities. Returns are annualised and constructed using pooled end-of-month data for six currencies—AUD, CAD, CHF, EUR, JPY and GBP—*vis-à-vis* the USD for different country samples spanning 1980:01-2017:12.

D Theoretical Derivations

D.1 Derivations for Section 5.2

Consider equations (22)-(23), repeated below for convenience:

$$m^{\mathbb{T}}_{t,t+1} = \log\beta - \frac{1}{2}\gamma\sigma^2_{\mathbb{T}} + \alpha_0\epsilon_{\mathbb{T},t+1} + \alpha_1\epsilon_{\mathbb{T},t} \underbrace{-\frac{1}{2}\delta\sigma^2_d + d_0\epsilon_{d,t+1} + d_1\epsilon_{d,t}}_{\text{unspanned transitory factor}}$$

$$m^{\mathbb{P}}_{t,t+1} = -\frac{1}{2}\sigma^2_{\mathbb{P}} + \epsilon_{\mathbb{P},t+1}$$

Construct $m^{\mathbb{T}}_{t,t+\kappa} = \sum_{i=0}^{\kappa-1} m^{\mathbb{T}}_{t+i,t+i+1}$, $m^{\mathbb{P}}_{t,t+\kappa} = \sum_{i=0}^{\kappa-1} m^{\mathbb{P}}_{t+i,t+i+1}$ as follows:

$$\begin{aligned} m^{\mathbb{T}}_{t,t+\kappa} &= \kappa\log\beta - \frac{1}{2}\gamma\kappa\sigma^2_{\mathbb{T}} + \alpha_0\epsilon_{\mathbb{T},t+1} + \alpha_1\epsilon_{\mathbb{T},t} + \cdots + \alpha_0\epsilon_{\mathbb{T},t+\kappa} + \alpha_1\epsilon_{\mathbb{T},t+\kappa-1} \\ &\quad -\frac{1}{2}\delta\kappa\sigma^2_d + d_0\epsilon_{d,t+1} + d_1\epsilon_{d,t} + \cdots + d_0\epsilon_{d,t+\kappa} + d_1\epsilon_{d,t+\kappa-1}, \qquad \text{(D1)} \\ m^{\mathbb{P}}_{t,t+\kappa} &= -\frac{1}{2}\kappa\sigma^2_{\mathbb{P}} + \epsilon^{\mathbb{P}}_{t+1}\cdots + \epsilon^{\mathbb{P}}_{t+\kappa} \qquad \text{(D2)} \end{aligned}$$

These imply the following moments:

$$\begin{aligned} \mathbb{E}_t[m^{\mathbb{T}}_{t,t+\kappa}] &= \kappa\log\beta - \frac{1}{2}\gamma\kappa\sigma^2_{\mathbb{T}} + \alpha_1\epsilon_{\mathbb{T},t} - \frac{1}{2}\delta\kappa\sigma^2_d + d_1\epsilon_{d,t}, \\ \mathbb{E}_t[m^{\mathbb{P}}_{t,t+\kappa}] &= -\frac{1}{2}\kappa\sigma^2_{\mathbb{P}}, \\ \text{var}_t(m^{\mathbb{T}}_{t,t+\kappa}) &= \alpha_0^2\sigma^2_{\mathbb{T}} + (\alpha_0+\alpha_1)^2(\kappa-1)\sigma^2_{\mathbb{T}} + d_0^2\sigma^2_d + (d_0+d_1)^2(\kappa-1)\sigma^2_d, \\ \text{var}_t(m^{\mathbb{P}}_{t,t+\kappa}) &= \kappa\sigma^2_{\mathbb{P}} \end{aligned}$$

Combining (10) with the above yields:

$$\begin{aligned} -r_{t,\kappa} &= \kappa\log\beta - \frac{1}{2}\gamma\kappa\sigma^2_{\mathbb{T}} - \frac{1}{2}\kappa\sigma^2_{\mathbb{P}} + \frac{1}{2}\left\{\alpha_0^2\sigma^2_{\mathbb{T}} + (\alpha_0+\alpha_1)^2(\kappa-1)\sigma^2_{\mathbb{T}}\right\} \qquad \text{(D3)} \\ &\quad +\frac{1}{2}\left\{d_0^2\sigma^2_d + (d_0+d_1)^2(\kappa-1)\sigma^2_d + \kappa\sigma^2_{\mathbb{P}}\right\}, \\ &= \kappa\log\beta + \frac{\sigma^2_{\mathbb{T}}}{2}\left(\alpha_0^2 - \gamma\kappa + (\alpha_0+\alpha_1)^2(\kappa-1)\right) + \frac{\sigma^2_d}{2}\left(d_0^2 - \gamma\kappa + (d_0+d_1)^2(\kappa-1)\right) \end{aligned}$$

Condition (24) then follows from $y_{t,\kappa} = \frac{1}{\kappa}r_{t,k}$.

D.1.1 Internationally Incomplete Markets

When markets are incomplete, we follow Backus et al. (2001) and Lustig and Verdelhan (2019) and consider a (log) exchange-rate process is given by $e_{t+1} - e_t = m_{t,t+1} - m^*_{t,t+1} + \eta_{t+1}$ where η_{t+1} is an incomplete-markets wedge.

Proposition D.1 (Term Structure and Market Incompleteness) *When international financial markets are incomplete ($\eta_{t+1} \neq 0$), relative yield-curve slopes are unaffected $cov_t(log(S^*_t)-$*

$\log(S_t), \eta_{t+1}) = 0$ *and* $proj(\mathbb{E}_t[rx^{FX}_{t+\kappa}] \mid \tilde{\epsilon})$ *is unchanged.*

Proof: Following Lustig and Verdelhan (2019), frictionless trade across borders in Home and Foreign bonds for $n = 1$ (i.e., (9) and (11)) and Foreign analogs, implies:

$$-\mathbb{E}_t[\eta_{t+1}] = \frac{1}{2}\text{var}_t(\eta_{t+1}) + \text{cov}_t(m_{t+1}, \eta_{t+1}), \tag{D4}$$

$$\mathbb{E}_t[\eta_{t+1}] = \frac{1}{2}\text{var}_t(\eta_{t+1}) + \text{cov}_t(m^*_{t+1}, -\eta_{t+1}) \tag{D5}$$

Evaluating (9), (11) and Foreign analogs for $r_t^{(n)}, r_t^{(n)\,*}$ for $n > 1$, then $\text{cov}_t(r_t^{(n)}, \eta_{t+1}) = 0\ \forall n$ which delivers the first part of the proposition. Moreover, additionally using (19) implies $\text{cov}_t(\tilde{\epsilon}, \eta_{t+1}) = 0$, delivering the second part. □

The proposition above does not depend on the specific form of SDFs and shows that, while market incompleteness will affect the volatility and predictability of exchange rates, it will not alter the relationship we study.[20]

One example of market incompleteness is the presence of convenience or liquidity yields. As shown in Engel and Wu (2022) and Jiang et al. (2024), these increase the predictability of exchange rates. However, Corsetti, Lloyd, and Marin (2020) shows convenience yields increase the predictability of the incomplete-markets wedge η_{t+1}, but are uncorrelated with the relative slope as long as they are constant along the term structure.

D.2 Derivations for Section 5.4

We focus on a symmetric model with country-specific factors.

Bond-Pricing Recursions. First, consider the one-period bond, $n = 1$:

$$\begin{aligned} p_{t,1} &= \mathbb{E}_t\left[m_{t,t+1}\right] + \frac{1}{2}\text{var}_t\left(m_{t,t+1}\right) \\ &= z_{0,t} + \left(1 - \frac{\lambda_1}{2}\right) z_{1,t} - \frac{\lambda_2}{2} z_{2,t} + \frac{\lambda_1^2}{2} z_{1,t} + \frac{\lambda_2^2}{2} z_{2,t} \\ &= -z_{0,t} + z_{1,t} \end{aligned}$$

where the first line uses the expression for the bond price for $n = 1$, the conditional expectation of equation (28) is used in the second line, and the resulting expression is rearranged to yield the third line. The one-period risk-free yield $y_{t,1}$ is therefore given by:

$$y_{t,1} = z_{0,t} - z_{1,t}$$

[20] Lustig and Verdelhan (2019) discuss that market incompleteness is an unlikely candidate for resolving exchange-rate puzzles, although Marin and Singh (2024) show that international market incompleteness has stronger implications in the presence of within country idiosyncratic risk.

Next, consider the general n-period bond price:

$$\begin{aligned}
p_{t,n} =& \mathbb{E}_t\left[m_{t,t+1} + p_{t+1,n-1}\right] + \frac{1}{2}\text{var}_t\left(m_{t,t+1} + p_{t+1,n-1}\right) \\
=& -z_{0,t} + \left(1 - \frac{1}{2}\lambda_1^2\right) z_{1,t} - \frac{1}{2}\lambda_2^2 z_{2,t} - \Omega_{n-1} - A_{n-1}z_{0,t+1} - B_{n-1}z_{1,t+1} - C_{n-1}z_{2,t+1} \\
&+ \frac{1}{2}\text{var}_t\left(\lambda_1\sqrt{z_{1,t}}u_{t+1} + \lambda_2\sqrt{z_{2,t}}u_{2,t+1} - A_{n-1}z_{0,t+1} - B_{n-1}z_{1,t+1} - C_{n-1}z_{2,t+1}\right) \\
=& -z_{0,t} + \left(1 - \frac{1}{2}\lambda_1^2\right) z_{1,t} - \frac{1}{2}\lambda_2^2 z_{2,t} - A_{n-1}[(1-\phi_0)z_{2,t} + \phi_0 z_{0,t}] \\
&- B_{n-1}[(1-\phi_1)z_{2,t} + \phi_1 z_{1,t}] - C_{n-1}[(1-\phi_2)\theta + \phi_2 z_{2,t}] \\
&+ \frac{1}{2}\left(\lambda_1^2 z_{1,t} + \lambda_2^2 z_{2,t} + A_{n-1}^2\sigma_0^2 z_{0,t} + B_{n-1}^2\sigma_1^2 z_{1,t} + C_{n-1}^2\sigma_2^2 z_{2,t} - \lambda_1 B_{n-1}\sigma_1 z_{1,t} - \lambda_2 C_{n-1}\sigma_2 z_{2,t}\right)
\end{aligned}$$

where the second line uses the linear pricing equation and (28); and the third line uses the process for the factors. Rearranging,the recursions can be seen in the final line:

$$\begin{aligned}
\Omega_n =& \Omega_{n-1} + C_{n-1}(1-\phi_2)\theta \\
A_n =& 1 + A_{n-1}\phi_0 - A_{n-1}^2\sigma_0^2 \\
B_n =& -1 + B_{n-1}\phi_1 + \frac{1}{2}\lambda_1\sigma_1 B_{n-1} - \frac{1}{2}\left(B_{n-1}\sigma_1\right)^2 \\
C_n =& A_{n-1}(1-\phi_0) + B_{n-1}(1-\phi_1) + \phi_2 C_{n-1} + \frac{1}{2}\lambda_2\sigma_2 C_{n-1} - \frac{1}{2}\left(C_{n-1}\sigma_2\right)^2
\end{aligned}$$

with initial conditions $A_0 = C_0 = 0$, $B_0 = -1$.

Bond Excess Returns. The *ex ante* n-period bond excess return is defined as $\mathbb{E}_t[rx_{t,t+1}^{(n)}] = \mathbb{E}_t[p_{t+1,n-1} - p_{t,n} - y_{t,1}]$. This can be written as:

$$\begin{aligned}
\mathbb{E}_t\left[rx_{t,t+1}^{(n)}\right] =& \mathbb{E}_t\left[p_{t+1,n-1} - p_{t,n} - y_{t,1}\right] \\
=& \mathbb{E}_t\left[-\Omega_{n-1} + \Omega_n - A_{n-1}z_{0,t+1} + A_n z_{0,t} - B_{n-1}z_{1,t+1} + B_n z_{1,t} - C_{n-1}z_{2,t+1} + C_n z_{2,t}\right] \\
&- z_{0,t} + z_{1,t} \\
=& C_{n-1}(1-\phi_2)\theta_2 - A_{n-1}\mathbb{E}_t[z_{0,t+1}] + A_n z_{0,t} - B_{n-1}\mathbb{E}_t[z_{1,t+1}] + B_n z_{1,t} \\
&- C_{n-1}\mathbb{E}_t[z_{2,t+1}] + C_n z_{2,t} - z_{0,t} + z_{1,t} \\
=& \left[-A_{n-1}\phi_0 + A_n - 1\right] z_{0,t} + \left[-B_{n-1}\phi_1 + B_n + 1\right] z_{1,t} + \\
&\left[-A_{n-1}(1-\phi_0) - B_{n-1}(1-\phi_1) - C_{n-1}\phi_2 + C_n\right] z_{2,t}
\end{aligned}$$

where line 2 uses the pricing equation, line 3 uses the recursion for A_n defined above, and line 4 expands the conditional expectation of factors and collects like terms. Evaluating the expression above in the limit as $n \to \infty$ yields:

$$\begin{aligned}
\mathbb{E}_t\left[rx_{t,t+1}^{(\infty)}\right] = & \left[A_\infty(1-\phi_0) - 1\right] z_{0,t} + \left[B_\infty(1-\phi_1) + 1\right] z_{1,t} + \\
& \left[(1-\phi_2)C_\infty - A_\infty(1-\phi_0) - B_\infty(1-\phi_1)\right] z_{2,t}
\end{aligned} \tag{D6}$$

which can be rearranged as:

$$\mathbb{E}_t\left[rx^{(\infty)}_{t,t+1}\right] = \quad [A_\infty(1-\phi_0)]\,(z_{0,t}-z_{2,t}) + [B_\infty(1-\phi_1)]\,(z_{1,t}-z_{2,t}) + [C_\infty(1-\phi_2)]\,z_{2,t} - z_{0,t} + z_{1,t} \tag{D7}$$

The bond premium is driven by the distance of factors 0 and 1 from their long-run mean and from movements in the long run mean itself. From the recursion formulas:

$$\mathbb{E}_t\left[rx^{(\infty)}_{t,t+1}\right] = \quad \left[-\frac{1}{2}\left(A_\infty\sigma_0\right)^2\right] z_{0,t} + \left[\frac{1}{2}\lambda_1\sigma_1 B_\infty - \frac{1}{2}\left(B_\infty\sigma_1\right)^2\right] z_{1,t} + \left[\frac{1}{2}\lambda_2\sigma_2 C_\infty - \frac{1}{2}\left(C_\infty\sigma_2\right)^2\right] z_{2,t}$$

Then, the *ex ante* bond risk premium is given by:

$$\begin{aligned}\mathbb{E}_t\left[rx^{(\infty)}_{t,t+1}\right] + \frac{1}{2}\mathrm{var}_t(r_{n,t+1}) &= -\,\mathrm{cov}_t(p_{t+1,n-1}, m_{t,t+1}) \\ &= -\,\lambda_1\sigma_1 B_{n-1} z_{1,t} - \lambda_2\sigma_2 C_{n-1} z_{2,t},\end{aligned}$$

recovering equation (29) in the main body. Factor zero does not appear because there is a zero price of risk.

Yield-Curve Slope and Bond Premium Approximation. The yield curve slope is defined as the difference between yields on n- and 1-period bonds:

$$S_{t,n} = y_{t,n} - y_{t,1} = \frac{1}{n}\left(\Omega_n + A_n + B_n z_{1,t} + C_n z_{2,t}\right) - z_{0,1} + z_{1,t}$$

Evaluating this expression in the limit as $n \to \infty$ yields:

$$S_{t,\infty} = C_\infty(1-\phi_2)\theta - z_{0,t} + z_{1,t}$$

which arises from the recursions for A_n, B_n and C_n, where A_n, B_n and C_n have a finite limit and Ω_n grows linearly. The approximation of the slope by the bond risk premium $S_{t,\infty} \approx \mathbb{E}_t[rx^{(\infty)}_{t,t+1}]$ is also verified within the CIR model. Over long enough samples, $\mathbb{E}_t[z_{0,t}] = \mathbb{E}_t[z_{1,t}] = \mathbb{E}_t[z_{2,t}] = \theta$, yielding the result.

Exchange Rates. Under complete markets, (log) one-period exchange rate changes are determined as:

$$\mathbb{E}_t[e_{t+1}] - e_t = \mathbb{E}_t[m_{t,t+1} - m^*_{t,t+1}] = -(z_{0,t} - z^*_{0,t}) + (1-\lambda_1^2/2)(z_{1,t} - z^*_{1,t}) + \lambda^2/2(z_{2,t} - z^*_{2,t})$$

We focus on conditional risk premia because our symmetric setup implies that unconditional risk premia $\mathbb{E}_t[rx^{FX}_{t+1}]$ are zero. The one-period ERRP can be derived by combining equations (13) and (28), assuming complete markets:

$$\mathbb{E}_t[rx^{FX}_{t+\kappa,t+\kappa+1}] = (\lambda_1^2/2)\mathbb{E}_t[z_{1,t+\kappa} - z^*_{1,t+\kappa}] + (\lambda_2^2/2)\mathbb{E}_t[z_{2,t+\kappa} - z^*_{2,t+\kappa}] \tag{D8}$$

where an increase in $z_{1,t+\kappa} - z^*_{1,t+\kappa}$ leads to a fall in $r_{t+\kappa} - r^*_{t+\kappa}$ and an increase in conditional ERRP, as in Fama (1984). However, the ERRP also depends on the second factor $(z_{2,t} - z^*_{2,t})$ so long maturity bonds are a useful orthogonal predictor, as in Ang and Chen (2010).

D.2.1 Predictability at Intermediate Horizons

This appendix establishes the ability of the CIR model to deliver orthogonal predictability of ERRP by bond premia at intermediate horizons.

Transitory-Permanent Variation. We begin by showing how to eliminate permanent innovations in the model, such that ERRP in the long-run are zero—and therefore there is no long-riun predictability. If there are no permanent innovations, Alvarez and Jermann (2005) show this requires:

$$\mathbb{E}_t\left[rx^{(\infty)}_{t,t+1}\right] = \frac{1}{2}\text{var}_t(m_{t,t+1})$$

Since this must be true for any value of $z_{0,t}, z_{1,t}, z_{2,t}$, using (D6), this requires:

$$\begin{aligned}
[A_\infty(1-\phi_0)-1] &= 0, && \text{(D9)}\\
[B_\infty(1-\phi_1)+1] &= \frac{1}{2}\lambda_1^2, && \text{(D10)}\\
[C_\infty(1-\phi_2)-B_\infty(1-\phi_1)-A_\infty(1-\phi_0)] &= \frac{1}{2}\lambda_2^2 && \text{(D11)}
\end{aligned}$$

such that:

$$\mathbb{E}_t\left[rx^{(\infty)}_{t+1} - rx^{(\infty)\,*}_{t+1}\right] = (\lambda_1^2/2)[z_{1,t}-z^*_{1,t}] + (\lambda_2^2/2)[z_{2,t}-z^*_{2,t}] \tag{D12}$$

coinciding exactly with the one-period $\mathbb{E}_t[rx^{FX}_{t+1}]$.

Longer-Horizon Currency Movements. The κ-step-ahead exchange-rate change is then given by:

$$\mathbb{E}_t[e_{t,t+\kappa}] - e_t = \sum_{i=1}^{\kappa}\mathbb{E}_t[\Delta^1 e_{t+i}] = \tag{D13}$$

$$\left[\underbrace{\frac{1-\phi_0^\kappa}{1-\phi_0}(z^*_{0,t}-z_{0,t}) + (1-\phi_0)\sum_{i=0}^{\kappa-1}\sum_{j=0}^{i-1}\phi_0^j(z^*_{2,t+i-1-j}-z_{2,t+i-1-j})}_{\sum_{i=1}^{\kappa-1}\mathbb{E}_t[z^*_{0,t+i}-z_{0,t+i}]}\right]$$

$$+\frac{\lambda_1^2-1}{2}\left[\underbrace{\frac{1-\phi_1^\kappa}{1-\phi_1}(z^*_{1,t}-z_{1,t}) + (1-\phi_1)\sum_{i=0}^{\kappa-1}\sum_{j=0}^{i-1}\phi_1^j(z^*_{2,t+i-1-j}-z_{2,t+i-1-j})}_{\sum_{i=1}^{\kappa-1}\mathbb{E}_t[z^*_{1,t+i}-z_{1,t+i}]}\right]$$

$$+\frac{\lambda_2^2}{2}\underbrace{\frac{1-\phi_2^k}{1-\phi_2}(z_{2,t}^*-z_{2,t})}_{\sum_{i=1}^{\kappa-1}\mathbb{E}_t[z_{2,t+i}^*-z_{2,t+i}]}$$

Importantly, expected depreciations are stricly increasing in both factors $(z_{1,t}, z_{2,t})$ ceteris paribus. This implies higher relevance of the second factor at longer horizons through two chanels. First, because factor 2 is assumed more persistent. Second, because factor 1 tend to factor 2 in the long run.

General Formulation with Hidden Factors. Finally, as in Section 5.2, we consider a generalization of the representative Home investor's SDF such that factor 2 can be hidden from longer maturity interest rates:

$$-m_{t,t+1} = z_{0,t} + (\xi + \lambda_1^2/2)z_{1,t} + \lambda_1\sqrt{z_{1,t}}\epsilon_{t+1} + (\zeta + \lambda_2^2/2)z_{2,t} + \lambda_2\sqrt{z_{2,t}}\epsilon_{2,t+1} \tag{D14}$$

The price of an $n-$period bond is given by

$$\begin{aligned} p_t^{(n)} &= -z_{0,t} - (\xi + \lambda_1^2/2)z_{1,t} - (\zeta_+\lambda_2^2/2)z_{2,t} - \Omega_{n-1} - A_{n-1}z_{0,t} - B_{n-1}z_{1,t} - C_{n-1}z_{2,t} \\ &+\frac{1}{2}\left\{\lambda_1^2 z_{1,t} + \lambda_2^2 z_{2,t} + A_{n-1}^2\sigma_0^2 z_{0,t} + B_{n-1}^2\sigma_1^2 z_{1,t} + C_{n-1}^2\sigma_2^2 z_{2,t} - \lambda_1\sigma_1 B_{n-1}z_{1,t} - \lambda_2\sigma_2 C_{n-1}z_{2,t}\right\} \end{aligned} \tag{D15}$$

It follows that:

$$\begin{aligned} \Omega_n =& \Omega_{n-1} + C_{n-1}(1-\phi_2)\theta \\ A_n =& 1 + A_{n-1}\phi_0 - A_{n-1}^2\sigma_0^2 \\ B_n =& \xi + B_{n-1}\phi_1 + \frac{1}{2}\lambda_1\sigma_1 B_{n-1} - \frac{1}{2}\left(B_{n-1}\sigma_1\right)^2 \\ C_n =& \zeta + A_{n-1}(1-\phi_0) + B_{n-1}(1-\phi_1) + \phi_2 C_{n-1} + \frac{1}{2}\lambda_2\sigma_2 C_{n-1} - \frac{1}{2}\left(C_{n-1}\sigma_2\right)^2 \end{aligned}$$

where $\Omega_0 = 0, A_0 = 1, B_0 = \xi, C_0 = \zeta$.

Consider, for example, the maturity $\kappa = 2$:

$$\begin{aligned} p_t^{(2)} &= -[\xi(1-\phi_2)\theta_2] - \left[(1+\phi_0) - \frac{1}{2}\sigma_0^2\right]z_{0,t} - \left[\xi(1+\phi_1) + \frac{1}{2}\lambda_1\xi\sigma_1 - \frac{1}{2}\xi^2\sigma_1^2\right]z_{1,t} \\ & - \left[(1-\phi_0) + \xi(1-\phi_1) - \zeta(1+\phi_2) - \frac{1}{2}\zeta^2\sigma_2^2 + \lambda_2\zeta\sigma_2\right]z_{2,t} \end{aligned} \tag{D16}$$

therefore,

$$p_t^{(2)} = \Omega_2 + A_2 z_{0,t} + B_2 z_{1,t}, \quad \text{if} \quad \zeta^2\sigma_2^2 + \zeta(1+\phi_2-\lambda_2\sigma_2) + [\xi(1-\phi_1) + (1-\phi_0)] = 0$$

which, once again imposes (20) and ensures factor $z_{2,t}$ is not reflected in the spot-yield differential.