Exchange Rates, Yield Curves and Transitory Risk*

Simon P. Lloyd† Emile A. Marin‡

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Abstract

We document that currencies with a steeper yield curve tend to depreciate against the US dollar, independently of interest-rate differentials, especially at 2 to 4-year horizons. Using survey data, we demonstrate that this relationship is driven by expectations of macroeconomic fundamentals reflected in the yield-curve slope. Within a no-arbitrage, preference-free framework, we derive conditions under which transitory risk can rationalize our documented relationship *alongside* the 'disconnect' of exchange rates from interest rates. We show these conditions emerge as equilibrium outcomes in models of domestically-incomplete markets.

JEL Codes: E43, F31, G12.

Key Words: Bond Yields; Exchange-Rate Risk Premia; Incomplete Markets; Term Structure; Uncovered Interest Parity.

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†Bank of England and Centre for Macroeconomics. Email Address: simon.lloyd@bankofengland.co.uk.

‡University of California, Davis. Email Address: emarin@ucdavis.edu.

1 Introduction

Uncovered interest parity (UIP) predicts that expected carry-trade returns should be zero. While UIP robustly fails at short horizons (Hansen and Hodrick, 1980; Fama, 1984; Engel, 2016), it cannot be rejected empirically over long horizons (Chinn and Meredith, 2005; Chinn and Quayyum, 2012) or with long-maturity bonds (Lustig, Stathopoulos, and Verdelhan, 2019). In this paper, we shift our focus to, comparatively underexplored, intermediate horizons and document how differences between long- and short-maturity interest rates—the yield-curve 'slope'—can explain exchange-rate dynamics, over and above spot-yield differentials. Our focus on the slope is motivated by its widespread use as a leading indicator of macroeconomic outcomes (Estrella and Hardouvelis, 1991; Estrella and Mishkin, 1998; Estrella, 2005; Bansal and Shaliastovich, 2013), allowing us to revisit the 'disconnect' between exchange rates and macroeconomic fundamentals (Meese and Rogoff, 1983), both empirically and theoretically.

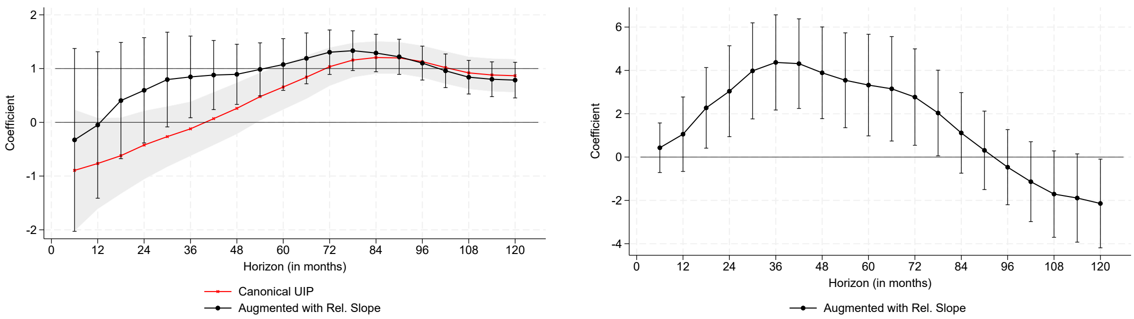
We first document an empirical association between relative yield-curve slopes and exchange-rates, which is strongest at medium-term (2 to 4-year) horizons. To provide an initial illustration of this novel finding, Figure 1 plots coefficient estimates from 'canonical' UIP regressions—of exchange-rate changes on spot-yield differentials—across horizons (from 6 months to 10 years) and UIP regressions 'augmented' with cross-country differences in yield-curve slopes—here, measured as the difference between observed 10-year and 6-month yields, a common proxy, for a panel of 6 advanced economies *vis-à-vis* the US.[1] Most striking is the significant tent-shaped relationship across horizons between the relative slope and exchange rates, conditional on interest-rate differentials (Figure 1b). At the 3-year peak, a 1pp increase in a country's yield-curve slope relative to the US is, on average, associated with a 4.4% cumulative (1.64% annualized) exchange-rate depreciation over that horizon.

We explore this result further by analyzing the association between relative yield-curve slopes, bond risk premia and exchange-rate risk premia (ERRP), extending the empirical analysis in Lustig et al. (2019). By distinguishing carry-trade strategies across holding periods and using different maturity bonds, our empirical framework has at least two advantages relative to the literature. First, we disentangle the relationship between the relative yield-curve slope and ERRP, as opposed to local-currency bond risk premia. Second, our specification reduces the empirical challenges posed by the limited number of non-overlapping observations in the long-horizon UIP regressions underpinning Figure 1. Still, as in the augmented UIP regression, we identify a tent-shaped relationship between exchange rates and relative slopes that peaks at 2 to 4-year holding periods, for a range of bond maturities, and is orthogonal to interest-rate differentials. We further show that the relationship is predominantly linked to ERRP. Our results are also robust to a range of specification changes, as well as the inclusion of liquidity yields (Du, Im, and Schreger, 2018; Engel and Wu, 2022) (the non-monetary returns that government bonds provide due to their safety, ease of resale, and collateral value).

[1] Figure 1a plots the estimated yield-differential coefficients. While our main focus is on Panel (b), the augmented regression does not challenge the common view that UIP can be rejected at short to medium horizons, but is harder to reject at longer horizons. Appendix A.2 details the regression specification.

Figure 1: Results from canonical and augmented UIP regressions

(a) Estimated relative-yield coefficients (b) Estimated relative-slope coefficients



Notes: Coefficient estimates from regressions of κ-month exchange-rate change on: (i) κ-month yield differentials; (ii) κ-month yield differentials and relative yield-curve slope. See Appendix A.2 for details. Panel (a): loading on relative-yield differentials from (i) (red crosses) and (ii) (black circles); (b): loading on relative-slope differentials from (ii). Horizontal axes plot horizons κ in months. Regressions estimated for 6 currencies (AUD, CAD, CHF, EUR, JPY, GBP) vs. USD, 1980:01-2024:09, pooled with country fixed effects. 95% confidence intervals, calculated using Driscoll and Kraay (1998) standard errors, denoted by gray shading/black bars.

To uncover the drivers of this relationship, we use survey data from *Consensus Economics* to show that the component of the relative yield-curve slope associated with ERRP is tightly linked to expectations of macroeconomic fundamentals. We do so by first isolating variation in relative yield-curve slopes explained by cross-country differences in expectations of GDP growth and inflation.[2] Going a step further, we show that it is predominantly this component of relative slopes—as opposed to the unexplained component—that generates the tent-shaped relationship between yield curves and exchange rates. So, our evidence suggests that the relationship between relative yield-curve slopes and intermediate-horizon exchange-rate dynamics that we uncover has its roots in cross-country differences in expected macroeconomic fundamentals.

Turning to theory, we show that the empirical association between relative slopes and ERRP can be reproduced within a no-arbitrage framework, *alongside* the 'disconnect' of exchange rates from spot yields. To do so, we build on the SDF process proposed in Alvarez and Jermann (2005), which expands on the canonical Vasicek (1977) model for the term-structure of interest rates, and undertake a risk-accounting exercise. The model incorporates three types of risk: (i) permanent innovations; (ii) transitory innovations; and (iii) transitory innovations which are restricted to have an offsetting conditional mean and variance. While all three drive exchange rates, (i) is not reflected in spot yields or yield-curves slopes (Chernov and Creal, 2023) and would result in failure of long-run UIP if not equalized across countries (Lustig et al., 2019). (ii) is reflected in both yields and slopes (Alvarez and Jermann, 2005). We show that only (iii) drives yield-curve slopes, but is 'hidden' from spot yields.

Through the lens of our decomposition, cross-country differences in both *transitory* inno-

[2] Bansal and Shaliastovich (2013) show that yield-curve slopes are associated with expectations of GDP growth and inflation within a country. Consistent with our focus on the *relative* price of currencies, our analysis hones in on *cross-country differences* in slopes and macroeconomic expectations.

vations drive exchange-rate dynamics. With transitory risk, booms—periods in which consumption has low marginal value—are expected to be followed by busts—periods in which consumption has high marginal value, and so yield curves slope upwards, on average, to compensate investors for risk (Piazzesi and Schneider, 2007). This 'business-cycle' risk is consistent with our empirical evidence using survey data showing that the association between ERRP and relative yield-curve slopes reflects cross-country differences in macroeconomic expectations. Our results echo Basu, Candian, Chahrour, and Valchev (2021) who identify a 'risk shock' that drives a large portion of aggregate co-movement over the business-cycle, including exchange rates, and contributes to a positive yield-curve slope. Furthermore, as long as transitory risk is predominantly due to the hidden factor (iii), this will also imply a disconnect of exchange rates from spot yields.

Not only are the conditions which underlie hidden factors plausible, they arise as *equilibrium* outcomes in a large class of models where heterogeneous agents trade in incomplete financial markets—e.g., due to segmentation within or across countries, or due to the presence of uninsurable idiosyncratic risk.[3] To see this, we consider a framework where there are two investors in the domestic economy. The marginal investor trades in both domestic and foreign bonds and is exposed to two (transitory) factors, while the other only invests in domestic bonds and is exposed only to one. Here, no arbitrage *requires* that the second factor be hidden from bond prices of any traded maturity (i.e., it be unspanned), since investors must agree on pricing, yet the second factor will still be reflected in the yield curve slope. This occurs because the hidden factor has offsetting effects on the mean and variance of SDFs. Our findings are therefore a challenge for general-equilibrium models which generate a functional relationship between the mean and variance of SDFs—notably, those with external habits (Campbell and Cochrane, 1999) and long-run risk (Bansal and Shaliastovich, 2013)—as summarized in Hassan, Mertens, and Wang (2024), who argue that this creates a tension between closed and open-economy asset pricing. To speak to these contributions, we conclude by showing our findings survive in a class of dynamic asset-pricing models that allow for time-varying volatility (e.g., Cox, Ingersoll, and Ross, 1985).

Related Literature. While previous studies have noted an empirical association between exchange rates and relative yield-curve slopes (e.g., Ang and Chen, 2010; Chen and Tsang, 2013; Gräb and Kostka, 2018; Chernov and Creal, 2020), our analysis goes further in three ways.[4] First, our focus is on intermediate horizons whereas existing studies focus on horizons of

[3] Within-country segmentation is studied in, e.g., Alvarez, Atkeson, and Kehoe (2009); Hassan (2013), cross-country segmentation in, e.g., Gabaix and Maggiori (2015); Itskhoki and Mukhin (2017); Chernov, Haddad, and Itskhoki (2024). Gourinchas, Ray, and Vayanos (2022) includes both. Incomplete markets due to idiosyncratic risk and the associated asset-pricing implications are studied in Constantinides and Duffie (1996a); Alvarez et al. (2009); Hassan (2013); Marin and Singh (2024).

[4] Chen and Tsang (2013) only find significance at short horizons, but we attribute this difference to the fact they capture relative yield-curve factors by directly estimating Nelson and Siegel (1987) decompositions from relative interest-rate differentials, thus assuming common factor structures across countries. In contrast, we construct proxies for factors using yield curves estimated on a country-by-country basis, allowing factor structures to be country-specific.

less than 2 years. Second, by extending the empirical setup of Lustig et al. (2019), we distinguish the relative slope's role for different components of bond returns to show a particular association between the slope and ERRP, not local-currency bond premia. Third, we use survey data to document new evidence that the relationship between the relative slope and ERRP is specifically associated with cross-country differences in macroeconomic expectations.

In doing so, we contribute more generally to the literature 'reconnecting' currency moves to fundamentals. While existing examples use, e.g., production data (Colacito, Riddiough, and Sarno, 2020), capital-flow data (Lilley, Maggiori, Neiman, and Schreger, 2022) or productivity-news shocks (Chahrour, Cormun, De Leo, Guerron-Quintana, and Valchev, 2021) to assess the macroeconomic origins of exchange-rate dynamics, our study connects exchange-rate changes to cross-country differences in macroeconomic expectations captured in survey data. To date, a limited literature has used such data to assess short-horizon exchange-rate anomalies (e.g., Candian and De Leo, 2023; Stavrakeva and Tang, 2023) but our analysis is unique in its particular focus on medium-term horizons.

Our theoretical exposition contributes to a literature using term-structure models to understand exchange-rate determination (e.g., Lustig et al., 2019; Chernov and Creal, 2023). We make three further contributions here. First, while others have shown an association between the relative slope and ERRP within a no-arbitrage setup, we introduce restrictions to transitory risk to reproduce the 'connection' between yield-curve slopes and ERRP *alongside* the 'disconnect' between ERRP and spot yields *and* the failure to reject long-horizon UIP. Second, while term-structure models are almost exclusively written under complete markets with a unique SDF, we contribute to a nascent literature on 'hidden' factors (Joslin, Priebsch, and Singleton, 2014; Bakshi, Crosby, Gao, and Hansen, 2023), illustrating how these restrictions arise as equilibrium outcomes in models of incomplete markets. Finally, we build on Balduzzi, Das, and Foresi (1998) and Ang and Chen (2010) to show how our results generalize to models of stochastic volatility. Relative to Ang and Chen (2010), we derive the conditions required to *jointly* deliver the failure of UIP, upward sloping yield curves and a relationship between yield curves and ERRP.

Outline. Section 2 presents our empirical results. Section 3 describes our preference-free, no-arbitrage framework, establishing a link between yield-curve slopes and transitory risk. Section 4 specifies a model which rationalizes our findings alongside the disconnect from short yields. Section 5 concludes.

2 Empirical Link Between Exchange Rates and Relative Slopes

We first present new empirical evidence about the link between exchange rates, *vis-à-vis* the US dollar, and relative yield-curve slopes. We then show that the relationship between relative slopes and ERRP at medium-term horizons is predominantly associated with cross-country differences expectations of GDP growth and inflation.

2.1 Excess Returns, Risk Premia and Relative Slopes

2.1.1 Empirical Setup

Let $P_{t,\kappa}$ denote the price of a κ-maturity zero-coupon bond at time t and $R_{t,\kappa} \geq 1$ denote the gross return on that bond. To decompose bond returns, we distinguish a bond's maturity κ from its holding period h, where $h \leq \kappa$ and $h = \kappa$ if and only if the bond is held to maturity. The h-month holding period return on a κ-month zero-coupon bond is $HPR_{t,t+h}^{(\kappa)} = P_{t+h,\kappa-h}/P_{t,\kappa}$ (the bond's resale price at $t+h$ when its maturity has diminished by h months relative to its time-t price). The (log) excess return on that bond over the holding period h is thus:

$$rx_{t,t+h}^{(\kappa)} = \log\left[\frac{HPR_{t,t+h}^{(\kappa)}}{R_{t,h}}\right] \tag{1}$$

where $R_{t,h}$ is the gross return on an h-month zero-coupon bond at time t, the risk-free rate.

The h-period (log) return on a Foreign bond, expressed in US dollars, in excess of the risk-free return in the base currency, $rx_{t,t+h}^{(\kappa),\$}$, can be decomposed as:

$$rx_{t,t+h}^{(\kappa),\$} = \log\left[\frac{HPR_{t,t+h}^{(\kappa)*}}{R_{t,h}}\frac{\mathcal{E}_t}{\mathcal{E}_{t+h}}\right] = \log\left[\frac{HPR_{t,t+h}^{(\kappa)*}}{R_{t,h}^*}\right] + \log\left[\frac{R_{t,h}^*}{R_{t,h}}\frac{\mathcal{E}_t}{\mathcal{E}_{t+h}}\right] = rx_{t,t+h}^{(\kappa)*} + rx_{t,t+h}^{FX} \tag{2}$$

where $rx_{t,t+h}^{(\kappa)*}$ represents the (log) local-currency bond return from a Foreign bond and $rx_{t,t+h}^{FX}$ is the (log) currency excess return.

To study the drivers of these returns, we first estimate the following panel regressions for different holding periods h and bond maturities κ:

$$\mathrm{y}_{j,t,h}^{(\kappa)} = \gamma_{2,h}^{(\kappa)}\left(S_{j,t}^* - S_t\right) + f_{j,h}^{(\kappa)} + \varepsilon_{j,t+h}^{(\kappa)} \tag{3}$$

where $S_{j,t}^*$ is the slope of the Foreign-country-j yield curve at time t, S_t is the slope of the base-country yield curve, [5] and $\mathrm{y}_{j,t,h}^{(\kappa)}$ is either:

- $rx_{j,t,t+h}^{(\kappa),\$} - rx_{US,t,t+h}^{(\kappa)}$: the *dollar bond-return difference*, the excess return on the Foreign bond in US dollar terms relative to the US return;
- $rx_{j,t,t+h}^{FX}$: the *exchange-rate risk premium (ERRP)*, the excess return from Foreign currency; or,
- $rx_{j,t,t+h}^{(\kappa)*} - rx_{US,t,t+h}^{(\kappa)}$: the *local-currency bond-return difference*, the excess return on the Foreign bond in Foreign-currency terms relative to the US return.

The coefficient $\gamma_{2,h}^{(\kappa)}$ captures the association between the relative slope and (annualized) ERRP or bond premia.

[5] Along with its curvature, the level and slope of the yield curve are known to capture a high degree of variation in bond yields (Litterman and Scheinkman, 1991).

Regression (3) aligns with the specification in Lustig et al. (2019) when $h = 1$—although they consider a 1-month holding period, while we look at 6-month holding periods, so comparison is not exact. Looking at $\kappa = 120$ only, they show that the relative yield-curve slope has an insignificant influence on dollar bond-return differences, but opposing effects on local-currency bond-return differences (positive coefficient) and ERRP (negative coefficient) that cancel out overall. Our empirical framework extends this, assessing the predictability of excess returns with yield-curve slope differentials at a range of maturities κ and holding periods h, bridging the gap between the canonical UIP regressions and Lustig et al. (2019).

To account for the contribution of the relative slope *over and above* spot-yield differentials, we also estimate the following extending regression:

$$\mathrm{y}_{j,t,h}^{(\kappa)} = \gamma_{1,h}^{(\kappa)} \left(r_{j,t,h}^{*} - r_{t,h} \right) + \gamma_{2,h}^{(\kappa)} \left(S_{j,t}^{*} - S_{t} \right) + f_{j,h}^{(\kappa)} + \varepsilon_{j,t+h}^{(\kappa)} \tag{4}$$

where the maturity of the relative spot yield, $r_{j,t,h} \equiv \log(R_{j,t,h})$, matches the holding period h of the excess return on the left-hand side $\mathrm{y}_{j,t,h}^{(\kappa)}$.

2.1.2 Data

We estimate these regressions using exchange- and interest-rate data for 7 jurisdictions with liquid bond markets: Australia, Canada, Switzerland, Euro Area, Japan, UK, and US. The US is the base country among our sample of G7 currencies. To capture the term structure of interest rates in each region, we use nominal zero-coupon government bond yields of 6, 12, 18, ..., 120-month maturities. Yield curves are obtained from a combination of sources, including central banks and Wright (2011) (Appendix A.1), so our bond-yield panel is unbalanced. Nominal exchange-rate data is from *Refinitiv*. We use end-of-month data from 1980:01 to 2024:09.

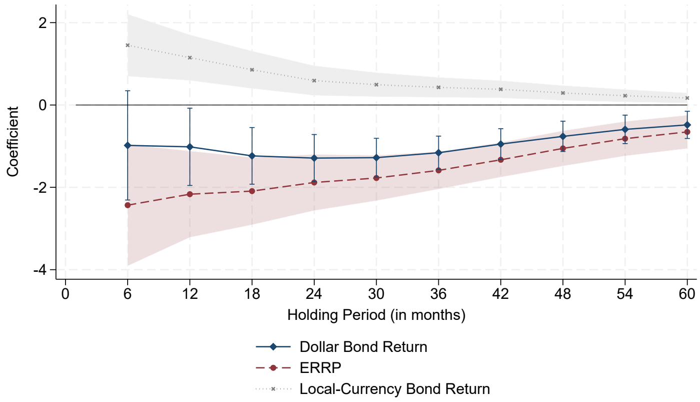
We measure the yield-curve slope in each region with proxies. We define the slope as the difference between the 10-year and 6-month yields, $S_{j,t}^{*} \equiv y_{j,t,10y}^{*} - y_{j,t,6m}^{*}$.[6] Since these proxies are constructed by taking cross-country differences derived from yield curves estimated on a country-by-country basis, we do not assume any symmetry in the factor structure of yield curves across countries—unlike Chen and Tsang (2013).

2.1.3 Results

Tables 1 and 2 present the full results for regression (3), with Figure 2 focusing on the coefficient estimates for the 10-year maturity only ($\kappa = 120$). Importantly, where our regression specification most closely matches Lustig et al. (2019), at short holding periods $h = 6$ and the longest maturity $\kappa = 120$, our results mirror theirs. The relative slope is insignificantly associated with the dollar bond-return difference (Panel A), a positive and significant influence on the local-currency bond-return difference (Panel C), and a negative and significant influence

[6] We prefer this proxy to principal-component estimates of the slope, which potentially contain look-ahead bias, being defined using weights estimated using information in the whole sample. By construction, our proxy is only based on information available up to time t. Nevertheless, our findings are robust to the use of principal-component-based measures.

Figure 2: Estimated relative slope coefficients from excess-return regressions across holding periods for 10-year maturity



Notes: $\hat{\gamma}_{2,h}^{(120)}$ estimates from regression (3) for dollar bond-return differences (blue diamonds), exchange-rate risk premia (maroon circles) and local-currency bond-return differences (grey crosses). Horizontal axis denotes the holding period h in months. Regressions estimated using pooled monthly data for 6 currencies (AUD, CAD, CHF, EUR, JPY, GBP) against the USD from 1980:01 to 2024:09, including country fixed effects. 95% confidence intervals, calculated using Driscoll and Kraay (1998) standard errors, denoted by shaded areas/bars around point estimates.

on the ERRP (Panel B). The latter two effects approximately cancel out for dollar bond-return differences. More generally, the short-horizon local-currency bond-return difference predictability confirm results for US bond returns (see, e.g., Fama and Bliss, 1987; Campbell and Shiller, 1991; Cochrane and Piazzesi, 2005).

Exploring our results across holding periods h and for all maturities κ, two observations are noteworthy. First, while the relative yield-curve slope does not significantly predict dollar bond-return differences at the 6-month holding period for 10-year bonds, the relative-slope loading for the same bond maturity is significantly non-zero over some longer holding periods. While, in the former case, the influence of the relative slope on currency and local-currency bond returns offset one another (in line with Lustig et al., 2019), our results indicate that the influence of the relative slope on the currency premium dominates over longer holding periods, even for long-term bonds. This does not necessarily contradict the conclusions in Lustig et al. (2019), but echoes that exchange rates may be near random-walk objects (e.g., Chernov and Creal, 2023; Andrews, Colacito, Croce, and Gavazzoni, 2024). Nevertheless, for a given holding period, the influence of the relative slope on dollar bond returns decreases in magnitude with maturity.

Second, for a given maturity, the loading on the relative slope tends to peak in magnitude at short-to-medium holding periods for the dollar bond risk premium. For the 66-month maturity, and above, the peak coefficient occurs at the 18-month holding period. Here a 1pp increase in the Foreign relative slope, *vis-à-vis* the US slope, is associated with a 1.2-1.8% annualised reduction in dollar-bond premia on Foreign bonds. This gives rise to a(n inverse) tent shaped relationship in the coefficients across h, as the Figure 2 shows for dollar bond-return differences for the

Table 1: Slope coefficient estimates from dollar bond-return and ERRP regressions

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
					Holding Periods h					
	6m	12m	18m	24m	30m	36m	42m	48m	54m	60m
Panel A: Dependent Variable $rx^{(\kappa),\$}_{j,t,t+h} - rx^{(\kappa)}_{US,t,t+h}$										
12m	-2.44***									
	(0.74)									
18m	-2.41***	-2.17***								
	(0.73)	(0.52)								
24m	-2.36***	-2.15***	-2.10***							
	(0.71)	(0.51)	(0.41)							
30m	-2.31***	-2.12***	-2.10***	-1.91***						
	(0.71)	(0.51)	(0.40)	(0.34)						
36m	-2.24***	-2.08***	-2.08***	-1.92***	-1.79***					
	(0.70)	(0.50)	(0.39)	(0.33)	(0.28)					
42m	-2.17***	-2.02***	-2.05***	-1.91***	-1.80***	-1.60***				
	(0.69)	(0.49)	(0.38)	(0.33)	(0.27)	(0.23)				
48m	-2.09***	-1.96***	-2.01***	-1.89***	-1.79***	-1.60***	-1.34***			
	(0.68)	(0.49)	(0.38)	(0.32)	(0.27)	(0.22)	(0.21)			
54m	-2.00***	-1.89***	-1.95***	-1.87***	-1.77***	-1.59***	-1.33***	-1.06***		
	(0.68)	(0.48)	(0.37)	(0.32)	(0.26)	(0.22)	(0.21)	(0.21)		
60m	-1.91***	-1.82***	-1.90***	-1.83***	-1.75***	-1.57***	-1.32***	-1.05***	-0.82***	
	(0.68)	(0.48)	(0.37)	(0.31)	(0.26)	(0.22)	(0.20)	(0.21)	(0.21)	
66m	-1.81***	-1.74***	-1.84***	-1.78***	-1.71***	-1.54***	-1.29***	-1.03***	-0.81***	-0.65***
	(0.68)	(0.48)	(0.36)	(0.31)	(0.25)	(0.21)	(0.20)	(0.21)	(0.21)	(0.20)
72m	-1.72**	-1.66***	-1.77***	-1.74***	-1.67***	-1.51***	-1.27***	-1.01***	-0.79***	-0.64***
	(0.68)	(0.48)	(0.36)	(0.31)	(0.25)	(0.21)	(0.20)	(0.20)	(0.20)	(0.20)
78m	-1.62**	-1.58***	-1.70***	-1.68***	-1.63***	-1.47***	-1.23***	-0.99***	-0.77***	-0.62***
	(0.68)	(0.48)	(0.36)	(0.30)	(0.25)	(0.21)	(0.20)	(0.20)	(0.20)	(0.19)
84m	-1.52**	-1.49***	-1.64***	-1.63***	-1.58***	-1.43***	-1.20***	-0.96***	-0.75***	-0.61***
	(0.68)	(0.48)	(0.36)	(0.30)	(0.25)	(0.21)	(0.19)	(0.20)	(0.19)	(0.19)
90m	-1.43**	-1.41***	-1.57***	-1.57***	-1.53***	-1.39***	-1.16***	-0.93***	-0.73***	-0.59***
	(0.68)	(0.48)	(0.36)	(0.30)	(0.25)	(0.21)	(0.19)	(0.20)	(0.19)	(0.19)
96m	-1.34**	-1.33***	-1.50***	-1.52***	-1.48***	-1.34***	-1.12***	-0.90***	-0.70***	-0.57***
	(0.68)	(0.48)	(0.36)	(0.30)	(0.24)	(0.21)	(0.19)	(0.19)	(0.19)	(0.18)
102m	-1.24*	-1.25***	-1.43***	-1.46***	-1.43***	-1.30***	-1.07***	-0.86***	-0.68***	-0.55***
	(0.68)	(0.48)	(0.35)	(0.30)	(0.24)	(0.21)	(0.19)	(0.19)	(0.18)	(0.18)
108m	-1.15*	-1.17**	-1.37***	-1.40***	-1.38***	-1.25***	-1.03***	-0.83***	-0.65***	-0.53***
	(0.68)	(0.48)	(0.35)	(0.30)	(0.24)	(0.21)	(0.19)	(0.19)	(0.18)	(0.18)
114m	-1.07	-1.09**	-1.30***	-1.35***	-1.33***	-1.20***	-0.99***	-0.80***	-0.62***	-0.51***
	(0.68)	(0.48)	(0.35)	(0.29)	(0.24)	(0.21)	(0.19)	(0.19)	(0.18)	(0.17)
120m	-0.98	-1.02**	-1.24***	-1.29***	-1.28***	-1.16***	-0.95***	-0.76***	-0.59***	-0.48***
	(0.68)	(0.48)	(0.35)	(0.29)	(0.24)	(0.21)	(0.19)	(0.19)	(0.18)	(0.17)
Panel B: Dependent Variable $rx^{FX}_{j,t,t+h}$										
S^R	-2.43***	-2.17***	-2.10***	-1.88***	-1.77***	-1.59***	-1.33***	-1.05***	-0.82***	-0.65***
	(0.76)	(0.54)	(0.42)	(0.35)	(0.28)	(0.23)	(0.22)	(0.22)	(0.22)	(0.21)

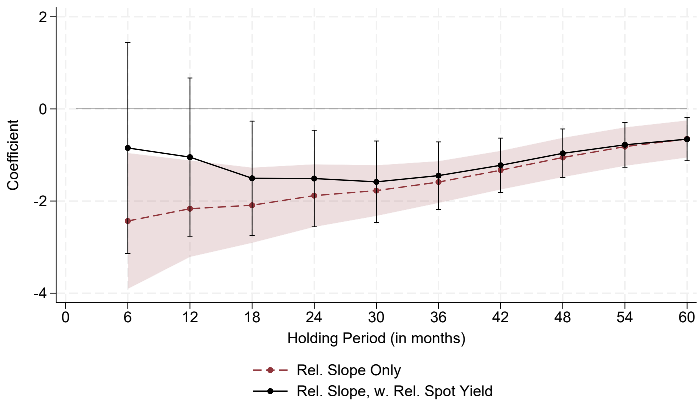
Notes: Coefficient estimates on the relative yield curve slope $S^R_t \equiv S^*_t - S_t$ from regressions with the (log) dollar bond-return difference (Panel A) or the (log) ERRP (Panel B) as dependent variables. Regressions estimated using pooled end-of-month data for 6 currencies (AUD, CAD, CHF, EUR, JPY, GBP) against the USD for 1980:01-2024:09. Log returns are annualised. All regressions include country fixed effects. The panels are unbalanced and Driscoll and Kraay (1998) standard errors are reported in parentheses. $*$, $**$ and $***$ denote significant point estimates at 10%, 5% and 1% levels, respectively.

Table 2: Slope coefficient estimates from local-currency bond-return regressions

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
	Holding Periods h									
	6m	12m	18m	24m	30m	36m	42m	48m	54m	60m
Panel C: Dependent Variable $rx^{(\kappa)}_{j,t,t+h} - rx^{(\kappa)}_{US,t,t+h}$										
12m	-0.01									
	(0.04)									
18m	0.02	-0.00								
	(0.08)	(0.03)								
24m	0.07	0.01	-0.01							
	(0.12)	(0.06)	(0.03)							
30m	0.13	0.04	-0.01	-0.03						
	(0.15)	(0.09)	(0.05)	(0.02)						
36m	0.19	0.09	0.01	-0.04	-0.02					
	(0.17)	(0.11)	(0.07)	(0.04)	(0.02)					
42m	0.26	0.14	0.04	-0.03	-0.03	-0.01				
	(0.20)	(0.12)	(0.09)	(0.06)	(0.03)	(0.01)				
48m	0.35	0.21	0.09	-0.01	-0.02	-0.02	-0.01			
	(0.22)	(0.15)	(0.11)	(0.07)	(0.05)	(0.03)	(0.01)			
54m	0.43*	0.28*	0.14	0.02	-0.00	-0.00	-0.00	-0.00		
	(0.24)	(0.16)	(0.12)	(0.08)	(0.06)	(0.04)	(0.02)	(0.01)		
60m	0.52**	0.35**	0.19	0.05	0.03	0.02	0.01	0.01	0.00	
	(0.26)	(0.17)	(0.13)	(0.10)	(0.07)	(0.05)	(0.03)	(0.02)	(0.01)	
66m	0.62**	0.43**	0.26*	0.10	0.06	0.05	0.04	0.02	0.01	0.01
	(0.28)	(0.19)	(0.15)	(0.11)	(0.08)	(0.06)	(0.04)	(0.03)	(0.02)	(0.01)
72m	0.72**	0.51**	0.32**	0.15	0.10	0.08	0.07	0.04	0.03	0.02
	(0.29)	(0.20)	(0.16)	(0.12)	(0.09)	(0.06)	(0.05)	(0.04)	(0.03)	(0.02)
78m	0.81***	0.59***	0.39**	0.20	0.15	0.12	0.10*	0.07	0.05	0.03
	(0.31)	(0.21)	(0.17)	(0.13)	(0.10)	(0.07)	(0.06)	(0.05)	(0.04)	(0.02)
84m	0.91***	0.67***	0.46**	0.25*	0.20*	0.16**	0.14**	0.09*	0.07	0.05
	(0.32)	(0.22)	(0.18)	(0.14)	(0.11)	(0.08)	(0.07)	(0.05)	(0.04)	(0.03)
90m	1.00***	0.76***	0.52***	0.31**	0.25**	0.20**	0.17**	0.13**	0.09*	0.06*
	(0.33)	(0.23)	(0.19)	(0.14)	(0.11)	(0.09)	(0.07)	(0.06)	(0.05)	(0.04)
96m	1.10***	0.84***	0.59***	0.37**	0.30**	0.25***	0.22***	0.16**	0.12**	0.08**
	(0.34)	(0.24)	(0.20)	(0.15)	(0.12)	(0.10)	(0.08)	(0.07)	(0.06)	(0.04)
102m	1.19***	0.92***	0.66***	0.42***	0.35***	0.29***	0.26***	0.19**	0.14**	0.11**
	(0.35)	(0.25)	(0.21)	(0.16)	(0.13)	(0.10)	(0.09)	(0.07)	(0.06)	(0.05)
108m	1.28***	1.00***	0.73***	0.48***	0.40***	0.34***	0.30***	0.23***	0.17**	0.13**
	(0.36)	(0.26)	(0.21)	(0.17)	(0.14)	(0.11)	(0.10)	(0.08)	(0.07)	(0.05)
114m	1.37***	1.07***	0.79***	0.54***	0.45***	0.39***	0.34***	0.26***	0.20***	0.15**
	(0.37)	(0.27)	(0.22)	(0.18)	(0.14)	(0.12)	(0.10)	(0.09)	(0.07)	(0.06)
120m	1.45***	1.15***	0.86***	0.59***	0.49***	0.43***	0.38***	0.29***	0.23***	0.17***
	(0.39)	(0.29)	(0.23)	(0.19)	(0.15)	(0.13)	(0.11)	(0.09)	(0.08)	(0.06)

Notes: Coefficient estimates on the relative yield curve slope $S^*_t - S_t$ from regressions with the (log) local-currency bond-return difference (Panel C) as dependent variable. Regressions estimated using pooled end-of-month data for 6 currencies (AUD, CAD, CHF, EUR, JPY, GBP) against the USD for 1980:01-2024:09. Log returns are annualised. All regressions include country fixed effects. The panels are unbalanced and Driscoll and Kraay (1998) standard errors are reported in parentheses. $*$, $**$ and $***$ denote significant point estimates at 10%, 5% and 1% levels, respectively.

Figure 3: Estimated relative slope coefficients from ERRP regressions across holding periods with and without controlling for relative spot-yield differentials



Notes: $\hat{\gamma}_{2,h}$ estimates from regressions (3) (maroon circles) and (4) (black circles) for exchange-rate risk premia. Horizontal axis denotes the holding period h in months. Regressions estimated using pooled monthly data for 6 currencies (AUD, CAD, CHF, EUR, JPY, GBP) against the USD from 1980:01 to 2024:09, including country fixed effects. 95% confidence intervals, calculated using Driscoll and Kraay (1998) standard errors, denoted by shaded areas/bars around point estimates.

10-year maturity. Although significant at shorter holding periods and longer maturities, the relative slope loadings are quantitatively small for local-currency bond premia. Panel B reveals that the relative slope is predominantly associated with ERRP, since loadings on currency excess returns dominate in explaining the relative slope's impact on relative dollar-bond risk premia.

The (inverse) tent-shaped relationship between the relative slope and excess returns arises when controlling for spot-yield differentials, as in regression (4). Figure 3 demonstrates this, presenting the coefficient estimates on the relative slope for ERRP from that regression, compared to the benchmark regression (3). The coefficient estimates are also tabulated in Panel A of Table 3. In the specification with spot-yield differentials as controls, the negative coefficient on the relative slope is significantly different from zero from the 18-month holding period and above. At the peak, 30-month horizon, a 1pp increase in the Foreign yield curve *vis-à-vis* the US is associated with a 1.6% annualised reduction in the Foreign ERRP—quantitatively consistent with the role of the relative slope in Figure (1). Therefore our results indicate that the relative slope has predictive power over and above spot-yield differences at business-cycle horizons specifically.

2.1.4 Robustness

Before proceeding, we briefly summarize the robustness of these empirical findings *vis-à-vis* the US dollar base. However, we concede that, like other UIP patterns, the tent-shaped relationship between relative slope and ERRP is specific to using the US dollar as the base currency, suggestive of a global 'dollar' factor, consistent with the analysis in, e.g., Jiang (2024).

Table 3: Robustness of relative slope coefficient estimates from regression (3) for $rx^{FX}_{t,t+h}$

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
	Holding Periods h									
	6m	12m	18m	24m	30m	36m	42m	48m	54m	60m
A: Controlling for interest-rate differentials										
S^R	-0.85	-1.05	-1.51**	-1.51***	-1.58***	-1.45***	-1.22***	-0.96***	-0.78***	-0.66***
	(1.17)	(0.88)	(0.63)	(0.54)	(0.45)	(0.37)	(0.30)	(0.27)	(0.25)	(0.24)
r^R_h	2.77	1.11	0.44	0.23	0.11	0.07	0.05	0.04	0.02	-0.00
	(1.74)	(0.70)	(0.37)	(0.25)	(0.18)	(0.13)	(0.09)	(0.07)	(0.06)	(0.05)
B: 1980:01-2005:12 sub-sample										
S^R	-1.06	-1.11	-1.63**	-1.69***	-1.77***	-1.65***	-1.50***	-1.29***	-1.15***	-1.05***
	(1.35)	(1.02)	(0.76)	(0.65)	(0.54)	(0.44)	(0.34)	(0.29)	(0.27)	(0.26)
r^R_h	4.90**	2.16**	0.98*	0.55	0.29	0.18	0.11	0.06	-0.00	-0.04
	(2.39)	(0.94)	(0.51)	(0.35)	(0.25)	(0.18)	(0.12)	(0.09)	(0.07)	(0.06)
C: 1995:01-2024:09 sub-sample										
S^R	-0.87	-1.05	-1.61***	-1.71***	-1.75***	-1.56***	-1.23***	-0.94***	-0.77**	-0.66*
	(1.11)	(0.81)	(0.61)	(0.53)	(0.45)	(0.40)	(0.35)	(0.36)	(0.37)	(0.36)
r^R_h	2.26	1.12*	0.44	0.18	0.06	0.06	0.07	0.06	0.04	0.01
	(1.84)	(0.62)	(0.34)	(0.24)	(0.17)	(0.12)	(0.10)	(0.09)	(0.08)	(0.07)
D: 1980:01-2019:12 sub-sample										
S^R	-0.75	-0.97	-1.46**	-1.42**	-1.50***	-1.39***	-1.17***	-0.93***	-0.77***	-0.66***
	(1.23)	(0.92)	(0.66)	(0.55)	(0.47)	(0.38)	(0.31)	(0.27)	(0.25)	(0.24)
r^R_h	3.27*	1.32*	0.53	0.31	0.15	0.10	0.08	0.06	0.02	-0.00
	(1.87)	(0.74)	(0.39)	(0.26)	(0.19)	(0.14)	(0.10)	(0.07)	(0.06)	(0.05)
E: Controlling for liquidity yields										
S^R	-0.80	-0.90	-1.32	-1.20	-1.37**	-1.37**	-0.99**	-0.61	-0.31	-0.10
	(2.05)	(1.31)	(0.85)	(0.73)	(0.63)	(0.56)	(0.49)	(0.48)	(0.44)	(0.39)
r^R_h	1.14	0.63	0.15	0.04	-0.02	-0.03	0.02	0.03	0.05	0.05
	(2.67)	(0.90)	(0.44)	(0.32)	(0.23)	(0.17)	(0.14)	(0.11)	(0.09)	(0.08)
η^R_{10y}	0.06	0.06*	0.05*	0.06**	0.06***	0.07***	0.07***	0.07***	0.07***	0.07***
	(0.04)	(0.03)	(0.03)	(0.02)	(0.02)	(0.02)	(0.01)	(0.01)	(0.01)	(0.01)

Notes: Coefficient estimates on the relative yield curve slope $S^R_t \equiv S^* - S$ from regressions with the (log) ERRP as dependent variable. Regressions estimated using pooled end-of-month data for 6 currencies (AUD, CAD, CHF, EUR, JPY, GBP) against the USD. Log returns are annualised. All regressions include country fixed effects. The panel is unbalanced and Driscoll and Kraay (1998) standard errors are reported in parentheses. $*$, $**$ and $***$ denote significant point estimates at 10%, 5% and 1% levels, respectively. Regressions include spot-yield differentials $r^R \equiv r^*_h - r_h$ as regressors. Regressions in Panel E additionally control for 10-year liquidity yields η^R_{10y} as regressors.

Sub-Sample Stability. Panels B, C and D of Table 3 demonstrate that the association between the relative slope and ERRP, against the US dollar, is robust to sub-sample splits. Panel B presents results for a sample ending prior to the global financial crisis, 1980:01-2005:12. Panel C shows results for a sample with the majority of time spanning the post-crisis period, 1995:01-2004:09. Panel D documents results for a pre-Covid sample, 1980:01-2019:12. In all cases, estimated coefficients peak in magnitude at the 30-month holding period.

Liquidity Yields. We also document that the significant relationship between the relative slope and ERRP at business-cycle horizons is robust to controlling for liquidity (or convenience) yields (i.e., non-pecuniary returns), which Engel and Wu (2022), Jiang et al. (2021), and others have shown are important for exchange-rate determination. To do this, we use data on the term structure of liquidity yields from Du et al. (2018). These measure the difference between risk-free market rates and government yields at different maturities to quantify the implicit yield on a government bond, correcting for other frictions in forward markets and sovereign risk. Let $\eta^R_{j,t,\kappa}$ denote the κ-horizon liquidity premium for a κ-horizon US government bond relative to an equivalent-maturity Foreign government bond yield in country j. An increase in $\eta^R_{j,t,\kappa}$ reflects an increase in the relative liquidity of US Treasuries *vis-à-vis* country j. With these measures, we extend regression (4) by estimating:[7]

$$\mathrm{y}_{j,t,h}^{(\kappa)} = \gamma_{1,h}^{(\kappa)} \left(r_{j,t,h}^{*} - r_{t,h}\right) + \gamma_{2,h}^{(\kappa)} \left(S_{j,t}^{*} - S_{t}\right) + \gamma_{3,h}^{(\kappa)} \eta_{j,t,\kappa} + f_{j,h}^{(\kappa)} + \varepsilon_{j,t+h}^{(\kappa)} \tag{5}$$

where $\mathrm{y}_{j,t,h}^{(\kappa)}$ has the same definition as in regression (3) and $\gamma_{2,h}^{(\kappa)}$ can be interpreted as the average influence of a 1pp increase in relative US Treasury convenience. When the ERRP is the dependent variable, we expect $\gamma_{2,h}^{(\kappa)}$ to be positive.

The results for current returns $rx_{t,t+h}^{FX}$ are shown for the 10-year liquidity yield η_{10y}^{R} in Panel E of Table 3. As before, the relative slope coefficient is significantly associated with ERRP at business-cycle holding periods—here 2.5 to 3.5 years. Corresponding investigation into the dollar bond-return differences confirms that the influence of the relative slope on dollar-bond returns predominantly works through ERRP.

Strikingly, the $\gamma_{3,h}^{(\kappa)}$ coefficients reveal a stronger association between liquidity yields and ERRP at longer horizons. The coefficients on relative liquidity yields rise monotonically with respect to holding periods and grow in significance. This complements existing studies into liquidity yields and exchange-rate dynamics (e.g., Engel and Wu, 2022; Jiang et al., 2021), which have focused on short-horizon returns.

2.2 Macro Expectations, Relative Slopes and ERRP

Next, we explore the roots of the relationship between cross-country differences in yield-curve slopes and ERRP. Motivated by widely studied links between *country-specific* yield-curve slopes and macroeconomic outcomes (e.g., Estrella and Hardouvelis, 1991; Estrella and Mishkin, 1998; Estrella, 2005), we investigate the relationship between *relative* yield-curve slopes across countries and *relative* business-cycle expectations using data from professional forecasters working at large financial institutions. We then show that the association between relative slopes and ERRP at medium-term horizons is specifically associated with cross-country differences in macroeconomic expectations.

2.2.1 Expectations and Relative Yield-Curve Slopes

We use forecasters' expectations for GDP growth and inflation from *Consensus Economics* for the period over which data is available for all G7 economies (1990:01-2022:12). The forecasts are formed for the current year ($y = 0$) and the next year ($y = 1$). We denote the average expectations of country-j forecasters for year-y GDP and inflation by $\overline{gdp}_{j,t}^{y,e}$ and $\overline{\pi}_{j,t}^{y,e}$, respectively. We also use data capturing uncertainty around forecasts, labelling the cross-sectional standard deviation of GDP and inflation expectations, across forecasters, by $\mathrm{std}(gdp^{y,e})_{j,t}$ and $\mathrm{std}(\pi^{y,e})_{j,t}$, respectively.

[7] Although the Du et al. (2018) data is available from 1991:04 for some countries and tenors, some series begin as late as 1999:01 due to data availability. All series end in 2021:03.

Table 4: Association between relative yield curve slope and relative business-cycle expectations

	(1)	(2)	(3)	(4)	(5)
Sample:		1990:01-2019:12			1990:01-2022:12
$\overline{gdp}^{0,e}$	-0.40***		-0.32***	-0.24***	-0.15**
	(0.09)		(0.09)	(0.08)	(0.07)
$\overline{gdp}^{1,e}$	0.57***		0.42***	0.37***	0.22*
	(0.16)		(0.16)	(0.14)	(0.11)
$\overline{cpi}^{0,e}$		-0.39***	-0.38***	-0.27***	-0.30***
		(0.10)	(0.11)	(0.09)	(0.08)
$\overline{cpi}^{1,e}$		-0.44***	-0.29**	-0.30**	-0.21
		(0.13)	(0.12)	(0.13)	(0.14)
std($gdp^{0,e}$)				-0.35	-0.17
				(0.48)	(0.36)
std($gdp^{1,e}$)				0.31	0.06
				(0.42)	(0.32)
std($cpi^{0,e}$)				0.97*	1.02**
				(0.50)	(0.44)
std($cpi^{1,e}$)				1.04**	1.40***
				(0.41)	(0.34)
Constant	-0.64***	-1.10***	-1.03***	-0.83***	-0.80***
	(0.17)	(0.15)	(0.18)	(0.16)	(0.14)
# Countries	6	6	6	6	6
Country FE	YES	YES	YES	YES	YES
Within R^2	0.107	0.159	0.220	0.216	0.194

Notes: Coefficient estimates from variants of regression (6), with the relative yield curve slope as the dependent variable. Regressions estimated using pooled end-of-month data for 6 countries (with currencies: AUD, CAD, CHF, EUR, JPY, GBP) against the US. All regressions include country fixed effects. Driscoll and Kraay (1998) standard errors are reported in parentheses. $*$, $**$ and $***$ denote significant point estimates at 10%, 5% and 1% levels, respectively.

We illustrate the link between relative business-cycle expectations and the relative yield-curve slope by estimating variants of the following regression:

$$S^*_{j,t} - S_t = \sum_{y=0,1} \Bigg[\vartheta_1 \left(\overline{gdp}^{y,e*}_{j,t} - \overline{gdp}^{y,e}_{US,t} \right) + \vartheta_2 \left(\overline{\pi}^{y,e*}_{j,t} - \overline{\pi}^{y,e}_{US,t} \right) + \vartheta_3 \left(\text{std}(gdp^{y,e*})_{j,t} - \text{std}(gdp^{y,e})_{US,t} \right) + \vartheta_4 \left(\text{std}(\pi^{y,e*})_{j,t} - \text{std}(\pi^{y,e})_{US,t} \right) \Bigg] + f_j + \epsilon_{j,t} \quad (6)$$

Table 4 presents the estimated coefficients. Given the marked swings in macroeconomic aggregates during the Covid-19 pandemic, columns (1)-(4) focus on a pre-Covid sample, 1990:01-2019:12—though results for the 1990:01-2022:12 sample, column (5), are similar. The coefficients on average expectations for GDP, for the current and next year, and current inflation are strongly significant in all specifications. The mean GDP-expectation coefficient changes sign across horizon. The coefficient on the current-year expectation indicates that relatively high near-term GDP-growth expectations are associated with a relatively flat yield curve—consistent with higher short-term rates in booms (e.g., Wachter, 2006). In contrast, relatively high expectations for future GDP growth are associated with a relatively steep yield curve—consistent with expectations of higher short-term interest rates in the future. The coefficients on mean inflation expectations are negative at both horizons, possibly indicating lower-frequency dynamics through price expectations: relatively high inflation expectations are associated with a relatively flat yield curve, consistent with a need for higher short-term interest rates to stave off persistence in the rate of price increases. In addition, column (4) highlights some role for uncertainty about inflation in yield curve slopes, with relatively high uncertainty associated with a relatively steep yield-curve slope—a feature that is consistent with inflation risk leading nominal bonds to command a term premium. All in all, the four specifications—plus the sample spanning Covid—demonstrate a strong association between asymmetries in business-cycle expectations and the relative yield-curve slope across countries, which is also consistent with theory.

2.2.2 Expectations and Excess Currency Returns

To reconnect movements in exchange rates to fundamentals, we test whether the component of the relative slope explained by business-cycle expectations is the main driver of ERRP dynamics at business-cycle horizons. To do so, we recover fitted values $\hat{S}^R_{j,t} \equiv \widehat{S^*_{j,t} - S_t}$ and residuals $\hat{\epsilon}_{j,t}$ from estimates of equation (6). We construct the fitted values and residuals by estimating equation (6) on a country-by-country basis, in order to account for cross-country heterogeneity. Pooling these estimates across countries, we then estimate variants of the following extension to regressions (3) and (4):

$$y^{(\kappa)}_{j,t,h} = \gamma^{(\kappa)}_{1,h}\left(r^*_{j,t,h} - r_{t,h}\right) + \gamma^{(\kappa)}_{2,h}\hat{S}^R_{j,t} + \gamma^{(\kappa)}_{3,h}\hat{\epsilon}_{j,t} + f^{(\kappa)}_{j,h} + \varepsilon^{(\kappa)}_{j,t+h} \tag{7}$$

where we replace the observed relative slope $S^*_{j,t} - S_t$ with the fitted value $\hat{S}^R_{j,t}$ and additionally include the residual $\hat{\epsilon}_{j,t}$, which captures the component of the relative slope which is unexplained by variation in macroeconomic expectations.[8]

Panel A of Table 5 presents the baseline results, from the regression including spot-yield differentials for the pre-Covid period 1990:01-2019:12. The coefficient on the fitted relative slope is significantly negative across similar holding periods to the regressions involving observed relative yield-curve slopes (4). This setup can be understood as a spanning regression, in the spirit of Joslin et al. (2014) indicating that the component of the relative slope explained by cross-country asymmetries in macroeconomic expectations has explanatory power for ERRP, orthogonal to interest rates. We investigate this relationship theoretically in Section 4.

Figure 4 plots the $\gamma_{2,h}$ and $\gamma_{3,h}$ estimates from that regression. While the coefficient estimates on the residual are insignificant across most holding periods in this specification, the fitted relative slope coefficient is significant at all but the 5-year horizon. For holding periods of a year or more, the coefficients, again, peak, at the 30-month horizon. This suggests that movements in the relative yield-curve slope attributable to changes in relative macroeconomic expectations explain variation in ERRP at these intermediate horizons, over and above

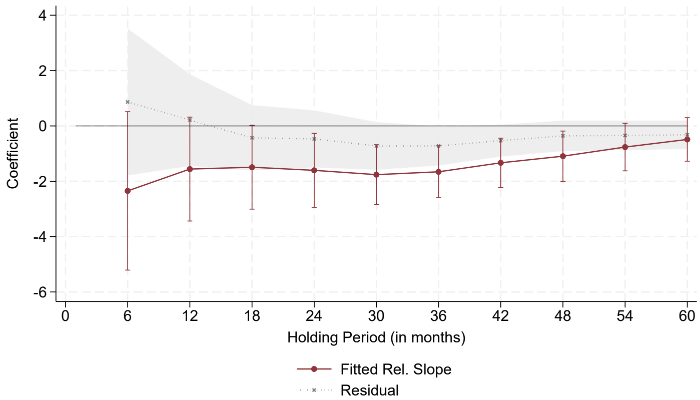
[8]The inclusion of the fitted residual $\hat{\epsilon}$ alongside the fitted value $\hat{S}^R$ from regression (6) additionally deals with concerns about inference with generated regressors. Pagan (1984) shows that consistent inference is possible with generated regressors when fitted values and residuals are used together in the same regression specification.

Table 5: Estimated relationship between ERRP and the component of relative slope driven by business-cycle expectations

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
	Holding Periods h									
	6m	12m	18m	24m	30m	36m	42m	48m	54m	60m
A: Controlling for relative interest-rate differentials (1990:01-2019:12)										
$\hat{S}^R$	-2.35	-1.56	-1.49*	-1.60**	-1.76***	-1.66***	-1.33***	-1.09**	-0.76*	-0.49
	(1.46)	(0.96)	(0.77)	(0.68)	(0.55)	(0.48)	(0.45)	(0.46)	(0.44)	(0.40)
$\hat{\epsilon}$	0.87	0.21	-0.43	-0.47	-0.72	-0.72**	-0.53*	-0.36	-0.34	-0.32
	(1.36)	(0.85)	(0.61)	(0.53)	(0.44)	(0.36)	(0.30)	(0.29)	(0.27)	(0.27)
r_h^R	3.33*	1.48**	0.74**	0.45*	0.23	0.17	0.15*	0.11	0.07	0.04
	(1.87)	(0.63)	(0.35)	(0.24)	(0.17)	(0.11)	(0.09)	(0.07)	(0.06)	(0.06)
B: Relative yield-curve slope terms only (1990:01-2019:12)										
$\hat{S}^R$	-4.13***	-2.96***	-2.42***	-2.28***	-2.15***	-1.96***	-1.62***	-1.32***	-0.91**	-0.57
	(1.36)	(0.97)	(0.81)	(0.73)	(0.60)	(0.50)	(0.47)	(0.47)	(0.44)	(0.40)
$\hat{\epsilon}$	-1.11	-1.39**	-1.52***	-1.27***	-1.18***	-1.09***	-0.88***	-0.64***	-0.53**	-0.42*
	(0.99)	(0.66)	(0.48)	(0.40)	(0.31)	(0.25)	(0.23)	(0.24)	(0.24)	(0.23)
C: Controlling for relative interest-rate differentials (1990:01-2022:12)										
$\hat{S}^R$	-2.41*	-1.71*	-1.61**	-1.71**	-1.84***	-1.69***	-1.36***	-1.10**	-0.78*	-0.51
	(1.45)	(0.99)	(0.78)	(0.69)	(0.55)	(0.47)	(0.44)	(0.45)	(0.43)	(0.39)
$\hat{\epsilon}$	0.56	0.02	-0.54	-0.66	-0.90**	-0.84**	-0.65**	-0.43	-0.35	-0.31
	(1.20)	(0.75)	(0.53)	(0.47)	(0.40)	(0.34)	(0.29)	(0.28)	(0.27)	(0.26)
r_h^R	2.61	1.22**	0.61**	0.35	0.16	0.12	0.11	0.09	0.07	0.04
	(1.69)	(0.56)	(0.31)	(0.22)	(0.15)	(0.11)	(0.08)	(0.07)	(0.06)	(0.06)

Notes: Coefficient estimates on the fitted relative yield curve slope $\hat{S}_t^R$ and residual $\hat{\epsilon}$, estimated from regression (6), with the (log) ERRP as dependent variable. Regressions estimated using pooled end-of-month data for 6 currencies (AUD, CAD, CHF, EUR, JPY, GBP) against the USD. Log returns are annualised. All regressions include country fixed effects. Driscoll and Kraay (1998) standard errors are reported in parentheses. $*$, $**$ and $***$ denote significant point estimates at 10%, 5% and 1% levels, respectively.

Figure 4: Estimated coefficients for fitted relative slope and residual from ERRP regressions across holding periods when controlling for relative spot-yield differentials



Notes: $\hat{\gamma}_{2,h}$ (maroon circles) and $\hat{\gamma}_{3,h}$ (grey crosses) estimates from regression (7) for exchange-rate risk premia. Horizontal axis denotes the holding period h in months. Regressions estimated using pooled monthly data for 6 currencies (AUD, CAD, CHF, EUR, JPY, GBP) against the USD from 1990:01 to 2019:12, including country fixed effects. 95% confidence intervals, calculated using Driscoll and Kraay (1998) standard errors, denoted by shaded areas/bars around point estimates.

spot-yield differentials.

Panels B and C of Table 5 report results from two supportive robustness exercises. Panel B reports coefficient estimates from regression (7) excluding spot-yield differentials. It continues to indicate that the fitted relative slope component is the predominant driver of ERRP. Panel

C reports estimates from regression (7) including spot-yield differentials for the full 1990:01-2022:12 sample. In spite of the inclusion of Covid-19 in the sample, where macroeconomic dynamics and their expectations were distinct from previous periods, variation in the relative slope fitted with macroeconomic expectations continue to act at the predominantly driver of ERRP—most strongly so at the 2.5-year horizon.

3 Preference-Free Theoretical Setup

Next, we present a no-arbitrage, preference-free framework and derive the conditions required to replicate the relationship between relative yields, relative yield-curve slopes and ERRP.

3.1 Pricing Kernels and Stochastic Discount Factors

The model has two countries: Home (base currency, US) and Foreign (denoted by an *), each populated by representative investor—which we later generalize in Section 4.2. The Home nominal pricing kernel V_t represents the marginal value of a currency unit at time t. No arbitrage implies the existence of a nominal SDF $M_{t,t+\kappa}$, which is given by the growth rate of the pricing kernel between periods t and $t+\kappa$: $M_{t,t+\kappa} = V_{t+\kappa}/V_t$. We make the following assumption about trade in bonds:

Assumption 1 *Investors in both countries can trade freely in Home- and Foreign-currency denominated risk-free bonds of maturities κ.*

The price of a Home zero-coupon bond that promises one currency unit κ periods into the future is given by: $P_{t,\kappa} = \mathbb{E}_t\left[M_{t,t+\kappa}\right] = \mathbb{E}_t\left[M_{t,t+1}P_{t+1,\kappa-1}\right]$, where $M_{t,t+1}$ denotes the one-period SDF and, by recursive substitution, $M_{t,t+\kappa} \equiv \prod_{i=0}^{\kappa-1} M_{t+i,t+i+1}$. Defining the gross return on a Home κ-period zero-coupon bond as $R_{t,\kappa} \equiv 1/P_{t,\kappa} \equiv (1+r_{t,\kappa}) \geq 1$, then:

$$\frac{1}{R_{t,\kappa}} = \mathbb{E}_t\left[M_{t,t+\kappa}\right] \tag{8}$$

which can be expanded as:

$$-r_{t,\kappa} = \mathbb{E}_t[m_{t,t+\kappa}] + \mathcal{L}_t(M_{t,t+\kappa}), \tag{9}$$

where $m_{t,t+\kappa} = \ln M_{t,t+\kappa}$ and $\mathcal{L}_t\left(M_{t,t+\kappa}\right) = \ln \mathbb{E}_t[M_{t,t+\kappa}] - \mathbb{E}_t[m_{t,t+\kappa}]$ denotes the conditional multi-period entropy of the SDF. If we assume one-period SDFs, $M_{t,t+1}^{(*)}$ are log-normally distributed, then (9) evaluated at $\kappa = 1$ is equivalent to: $\frac{1}{2}\text{var}_t\left(m_{t,t+1}\right) = L_t(M_{t,t+1})$. However, multi-period SDFs ($\kappa > 1$) will generally not be log-normally distributed if risk is heteroskedastic. Foreign expressions are analogously derived.

Based on (9), the following assumption delivers stable risk-free rates:

Assumption 2 *Stochastic discount factors $\frac{\Lambda_{t+1}}{\Lambda_t}$ and $\frac{\Lambda^*_{t+1}}{\Lambda^*_t}$ are jointly stationary.*

3.2 Transitory Risks and the Yield Curve

To understand sources of risk in the yield curve, we then use the following Alvarez and Jermann (2005) decomposition of the pricing kernel V_t into a permanent component $V_t^{\mathbb{P}}$ and a transitory component $V_t^{\mathbb{T}}$:

$$V_t = V_t^{\mathbb{P}} V_t^{\mathbb{T}}, \quad \text{where } V_t^{\mathbb{T}} = \lim_{\kappa \to \infty} \frac{\delta^{t+\kappa}}{P_{t,\kappa}} \tag{10}$$

where the constant δ is chosen to satisfy the regularity condition: $0 < \lim_{\kappa\to\infty} P_{t,\kappa}/\delta^{\kappa} < \infty$ for all t. A pricing kernel V_t is defined as having only transitory innovations if $\lim_{\kappa\to\infty} \frac{\mathbb{E}_{t+1}[V_{t+\kappa}]}{\mathbb{E}_t[V_{t+\kappa}]} = 1$. So, its permanent component follows a martingale, defined by: $V_t^{\mathbb{P}} = \lim_{\kappa\to\infty} \frac{\mathbb{E}_t[V_{t+\kappa}]}{\delta^{t+\kappa}}$. Importantly, the infinite-maturity bond can be written as a function of transitory innovations to SDFs only: $R_{t,\infty} = \lim_{\kappa\to\infty} R_{t,\kappa} = V_t^{\mathbb{T}}/V_{t+1}^{\mathbb{T}} = 1/M_{t,t+1}^{\mathbb{T}} = \exp(-m_{t,t+1}^{\mathbb{T}})$, where $m_{t,t+1}^{\mathbb{T}}$ denotes the transitory component of the SDF. In contrast, one-period bond returns, defined by equation (8), depend on both transitory and permanent innovations to SDFs.

More generally, *only* transitory risk is reflected across the term structure of interest rates. Define the (log) excess return from buying a n-period Home bond at time t for price $P_{t,n} = 1/R_{t,n}$ and selling it at time $t+1$ for $P_{t+1,n-1} = 1/R_{t+1,n-1}$ as $rx_{t,t+1}^{(n)} = p_{t+1,n-1} - p_{t,n} - y_{t,1}$, where $p_{t,n} \equiv \log(P_{t,n})$ and $y_{t,n} \equiv -\frac{1}{n} p_{t,n} \equiv \frac{1}{n} r_{t,n}$ is the annualized yield on a n-period bond. Assuming, for convenience, SDFs and prices are jointly log-normally distributed, this excess return can be written as:

$$\mathbb{E}_t\left[rx_{t,t+1}^{(n)}\right] + \frac{1}{2}\text{var}_t\left(r_{t+1,n}\right) = -\text{cov}_t\left(m_{t,t+1}^{\mathbb{T}}, \mathbb{E}_{t+1}\sum_{i=1}^{n-1} m_{t+i,t+i+1}^{\mathbb{T}}\right) \tag{11}$$

Over long enough samples, this risk premium is approximately equal to the yield-curve slope, $\mathbb{E}_t[rx_{t,t+1}^{(n)}] \approx S_t$ where $S_t \equiv y_{t,n} - y_{t,1}$, implying that the yield curve will be upward sloping on average if the covariance term is negative (Piazzesi and Schneider, 2007).[9]

The bond premium is positive if today's one-period SDF is negatively correlated with expected future marginal utility, consistent with a notion of transitory 'business-cycle' risk. That is, if households receive relatively good news about the distant future, they expect to value consumption less at long horizons (i.e., lower $\mathbb{E}_t[m_{t+i,t+i+1}]$ for some $i > 0$), but relatively highly in the near term (i.e., higher $m_{t,t+1}$). It follows that the relative yield-curve slope $S_t^* - S_t$ captures asymmetry or asynchronicity in transitory business-cycle risk across countries.

3.3 Exchange Rates and Currency Risk Premia

Finally, we explore how the sources of risk reflected in the yield curve impact exchange rates. Define the exchange rate $\mathcal{E}_t$ as the Foreign price of a unit of Home currency such that an increase corresponds to a Foreign depreciation. When engaging in cross-border asset trade, the Euler

[9] To derive this, re-write the excess return $rx_{t,t+1}^{(n)}$ as: $p_{t+1,n-1} - p_{t,n} - y_{t,1} = n y_{t,n} - (n-1) y_{t+1,n-1} - y_{t,1} = y_{t,n} - y_{t,1} - (n-1)(y_{t+1,n-1} - y_{t,n})$. Over a long enough sample and with large n, the difference between the average $(n-1)$-period yield and the average n-period yield is zero, implying that $\mathbb{E}_t[rx_{t,t+1}^{(n)}] \approx y_{t,n} - y_{t,1} \equiv S_t$.

equation for a Home investor holding a κ-period Foreign currency-denominated bond is:

$$1 = \mathbb{E}_t \left[M_{t,t+\kappa} \frac{\mathcal{E}_t}{\mathcal{E}_{t+\kappa}} R^*_{t,\kappa} \right] \tag{12}$$

By no arbitrage, the change in the nominal exchange rate corresponds to the ratio of SDFs:

$$\frac{\mathcal{E}_{t+\kappa}}{\mathcal{E}_t} = \frac{M_{t,t+\kappa}}{M^*_{t,t+\kappa}} e^{\eta_{t,t+\kappa}} \tag{13}$$

for all $\kappa > 0$, where $\eta_{t,t+\kappa}$ is the log incomplete-markets wedge as defined in Backus et al. (2001), such that $\eta_{t,t+\kappa} = 0$ characterizes complete markets. For simplicity, we assume internationally complete markets, but Appendix B.2.1 shows this does not drive our results. In the complete-markets case, equation (13) shows that the stationarity of exchange-rate changes follows from the stationarity of SDFs.

The (log) κ-period *ex-ante* currency risk premium $\mathbb{E}_t[rx^{FX}_{t,t+\kappa}]$ can be written as the difference in entropy of the Home and Foreign SDFs:

$$\begin{aligned} \frac{1}{\kappa}\mathbb{E}_t\left[rx^{FX}_{t,t+\kappa}\right] &= \frac{1}{\kappa}\left(r^*_{t,\kappa} - r_{t,\kappa} - \mathbb{E}_t\left[e_{t+\kappa}\right] - e_t + \mathbb{E}_t[\eta_{t,t+\kappa}]\right) \\ &= \frac{1}{\kappa}\left(\mathcal{L}_t\left(M_{t,t+\kappa}\right) - \mathcal{L}_t\left(M^*_{t,t+\kappa}\right)\right) \end{aligned} \tag{14}$$

This is the multi-period generalization of the one-period UIP return (e.g., Engel, 2016).

Lustig et al. (2019) show the failure to reject long-horizon UIP implies the equalization of the entropy of permanent SDF components. We formalize this assumption as:

Assumption 3 *The entropy of the permanent components of SDFs is roughly equal across countries* $\mathcal{L}_t\left(M^{\mathbb{P}}_{t,t+\kappa}\right) \approx \mathcal{L}_t\left(M^{\mathbb{P}\,*}_{t,t+\kappa}\right)$.

Imposing this assumption, then (15) implies:

$$\frac{1}{\kappa}\mathbb{E}_t\left[rx^{FX}_{t,t+\kappa}\right] \approx \frac{1}{\kappa}\left(\mathcal{L}_t\left(M^{\mathbb{T}}_{t,t+\kappa}\right) - \mathcal{L}_t\left(M^{\mathbb{T}\,*}_{t,t+\kappa}\right)\right) \tag{15}$$

Consequently, short and medium-horizon ERRP must reflect cross-country differences in the volatility of transitory innovations to SDFs resulting in a clear theoretical association between relative yield-curve slopes and ERRP, operating through transitory risk.

4 Implications for International Asset Pricing and 'Disconnect'

Having established that the association between yield-curve slopes and ERRP can be rationalized within the preference-free no-arbitrage framework laid out in Section 3, we investigate the broader implications of this relationship for international asset pricing debates around the 'disconnect' of exchange rates from fundamentals.

To make progress on this, we derive the restrictions implied on SDFs by our empirical results.

First, we define innovations $\tilde{\epsilon}$ such that:

$$\tilde{\epsilon} = \left\{\epsilon : \text{proj}(S_t^* - S_t|\epsilon) < 0 \;\&\; \text{proj}(r_{t,\kappa}^* - r_{t,\kappa}|\epsilon) = 0 \text{ for some } \kappa \geq 1\right\} \tag{16}$$

That is, following an increase in $\tilde{\epsilon}$, the Home slope becomes relatively steeper but interest-rate differentials of some maturity κ are unchanged. As long as Home households invest frictionlessly in the Home bond (9), per Assumption 1, this implies that movements in the conditional mean and variance of SDFs must perfectly offset for these innovations to not drive κ-maturity interest rates:[10]

$$\text{proj}(\mathbb{E}_t[m_{t,t+\kappa}]\;|\tilde{\epsilon}_t) = -\text{proj}(\mathcal{L}_t(M_{t,t+\kappa})\;|\tilde{\epsilon}_t) \tag{17}$$

Consistent with equation (11), these innovations must drive the autocovariance of the SDF in order to be reflected in the yield-curve slope and we know from our regressions that they are relevant for ERRP determination, $proj(\mathbb{E}_t[rx_{t+\kappa}^{FX}] > 0)$. In Section 4.2, we address the natural question of whether such restriction are plausible. We argue these restrictions are not only reasonable, but they necessarily emerge as equilibrium outcomes in a large class of incomplete-markets models.

4.1 A Stylized Model of Interest Rates and Exchange Rates

We specify a minimal model for exchange rates and the term structure of interest rates to highlight the role of transitory, yet unspanned, innovations to SDFs, and show how they map to the innovations detailed in (16). Our setup builds on Backus, Chernov, and Zin (2014) who consider a single transitory risk factor (denoted by $\mathbb{T}$) that is fully spanned by interest rates, resembling a truncated Vasicek (1977) model. Relative to this, we extend the model with two unspanned factors, a permanent factor ($\mathbb{P}$) which drives exchange-rate volatility, but is not reflected in the term structure of interest rates (Alvarez and Jermann, 2005; Chernov and Creal, 2023), and a novel unspanned-transitory factor (d) which is necessary for explaining the relationship between ERRP and the relative slope.

We do not take a stance on the specific interpretation of these factors. Rather we carry out a risk-accounting exercise to assess which factors explain which empirical regularities. Nevertheless, our evidence in Section 2.2 suggests that the transitory innovations are closely related to expectations of future macroeconomic fundamentals. Additionally, although this initial model is static model, it still allows us to map average empirical evidence at different horizons. Later, Section 4.3 demonstrates how the results generalize to a setting with time-varying volatility.

Let the (log) one-period SDF of the representative Home investor be decomposed as:

$$m_{t,t+1} = m_{t,t+1}^{\mathbb{T}} + m_{t,t+1}^{\mathbb{P}} \tag{18}$$

We assume the (truncated) Wold decompositions of the transitory and permanent components

[10] Hassan, Mertens, and Wang (2024) emphasize the tension between models that generate a negative functional relationship between the mean and variance of SDFs and the unpredictability of exchange rates. Given that we are specifically focusing on the predictable component, orthogonal to short rates, our results are consistent.

can, respectively, be written as:

$$m_{t,t+1}^{\mathbb{T}} = \log\beta - \frac{1}{2}\gamma\sigma_{\mathbb{T}}^2 + \alpha_0\epsilon_{\mathbb{T},t+1} + \alpha_1\epsilon_{\mathbb{T},t} \underbrace{-\frac{1}{2}\delta\sigma_d^2 + d_0\epsilon_{d,t+1} + d_1\epsilon_{d,t}}_{\text{unspanned transitory factor}} \tag{19}$$

$$m_{t,t+1}^{\mathbb{P}} = -\frac{1}{2}\sigma_{\mathbb{P}}^2 + \epsilon_{\mathbb{P},t+1} \tag{20}$$

where $\epsilon_{i,t}$ denote shocks to risk factors $i = \mathbb{T}, d, \mathbb{P}$ which we assume are i.i.d. mean zero, have constant variance σ_i^2 and are normally distributed. Foreign variables are defined analogously and denoted with asterisks.

In our risk-accounting exercise, we analyze which factor volatilities (i.e., σ_i^2 for $i = \mathbb{T}, d, \mathbb{P}$) are reflected in exchange rates, (relative) yields and the (relative) yield-curve slope. The key results rely only on Assumptions 1 and 2.[11] Per Assumption 2, we assume SDFs are stationary, and, as in the supplement to Alvarez and Jermann (2005), we set the mean of the (log) permanent component ($-\frac{1}{2}\sigma_{\mathbb{P}}^2$) to be non-zero and counteract the variance. The transitory component can be unrestricted (i.e., γ, δ need not equal to 1). We focus on a setting with symmetric factor loadings $\alpha_i = \alpha_i^*$, but allow for asymmetry in factor volatilities (i.e., $\sigma_i \neq \sigma_i^*$ for $i = \mathbb{T}, d, \mathbb{P}$).

Combining (9) with (18), the κ-period bond yield can be written as a function of both transitory factors' volatilities, $\sigma_{\mathbb{T}}$ and σ_d:

$$\begin{aligned} -y_{t,\kappa} = &\log\beta + \frac{1}{\kappa}\left[(d_0^2 - \kappa\delta) + (d_1 + d_0)^2(\kappa - 1)\right]\ \frac{\sigma_d^2}{2} \\ &+ \frac{1}{\kappa}\left[(\alpha_0^2 - \kappa\gamma) + (\alpha_1 + \alpha_0)^2(\kappa - 1)\right]\ \frac{\sigma_{\mathbb{T}}^2}{2} \end{aligned} \tag{21}$$

Critically, imposing Assumption 1 such that the SDF restriction (17) applies for $\kappa = 1$, we require $\delta = d_0^2$. With this, the short-term interest rate $r_t = y_{t,1}$ is a function of only the spanned-transitory factor volatility and is 'disconnected' from the unspanned-transitory factor volatility. Furthermore, the relationship with its volatility $\sigma_{\mathbb{T}}$ will be negative as long as $\alpha_0^2 - \gamma > 0$ (i.e., when precautionary-savings motives dominate). This is consistent with models of habits and long-run risk (e.g., Engel, 2016) that generate the correct sign for UIP deviations—since excess returns to Foreign currency are positive when Home volatility is high (Verdelhan, 2010).

However, the volatility of the unspanned-transitory factor, σ_d, will still be captured in the yield-curve slope, $S_t^{(\kappa)} = y_t^{(\kappa)} - r_t$:

$$S_t = \left(1 - \frac{1}{\kappa}\right)\left[d_0^2 - (d_1 + d_0)^2\right]\ \frac{\sigma_d^2}{2} + \left(1 - \frac{1}{\kappa}\right)\left[\alpha_0^2 - (\alpha_1 + \alpha_0)^2\right]\ \frac{\sigma_{\mathbb{T}}^2}{2} \tag{22}$$

Evaluating (14), the κ-horizon ERRP reflects asymmetries across countries in all three

[11] We do not require Assumption 3 which would further impose $\alpha_0 = -\alpha_1$ and $d_0 = -d_1$, but our results are robust to this. To see this, note that in the absence of permanent risk, the bond premium must equal half the variance of the SDF.

factors:

$$\mathbb{E}_t[rx^{FX}_{t+\kappa}] = \frac{1}{\kappa}\left[d_0^2 + (d_1 + d_0)^2(\kappa - 1)\right]\left(\frac{\sigma_d^2}{2} - \frac{\sigma_d^{*\,2}}{2}\right) + \frac{1}{\kappa}\left[\alpha_0^2 + (\alpha_1 + \alpha_0)^2(\kappa - 1)\right]\left(\frac{\sigma_{\mathbb{T}}^2}{2} - \frac{\sigma_{\mathbb{T}}^{*\,2}}{2}\right) + \left(\frac{\sigma_{\mathbb{P}}^2}{2} - \frac{\sigma_{\mathbb{P}}^{*\,2}}{2}\right) \quad (23)$$

Since the relative slope will reflect cross-country asymmetries in the volatilities of *both* transitory factors, this leaves scope for the relative slope to have explanatory power for ERRP *over and above* the short rate per our empirical findings. The following proposition then summarizes this key result:

Proposition 1 (Risk, Bond Yields and ERRP) *An increase in the factor volatilities $\sigma_{\mathbb{i}}$ for $i \in \{\mathbb{T}, \mathbb{P}\}$ are both associated with a higher ERRP $\frac{d\mathbb{E}_t[rx^{FX}_{t+\kappa}]}{d\sigma_i} > 0$, for $\kappa < \infty$, but cannot be associated with a relatively steep yield-curve slope $\frac{dS_t}{d\sigma_i} > 0$ without also being reflected in short interest-rate differentials $\frac{dy_{t,1}}{d\sigma_i} > 0$. In contrast, the volatility of the hidden factor (σ_d) can match these facts if and only if $\delta = d_0^2$.*

Proof: See Appendix B.1. □

The proposition above shows that only σ_d can satisfy the restriction for SDF dynamics (17), such that there are movements in the relative slope which are orthogonal to short yields. Put differently, only $\sigma_d \in \tilde{\epsilon}$ (see (16), emphasizing the role of these innovations for exchange-rate dynamics. More generally, our spanning regression (7) implies that σ_d reflects macroeconomic expectations and, if so, the model delivers a disconnect from short yields, but not a disconnect from fundamentals which are captured by the relative slope.

We can further extend this to rationalize the differential association between relative slope and ERRP across horizons. We consider an illustrative three-horizon case with a short-, medium- and long-run, where the orthogonal relationship between ERRP and relative yield-curve slope occurs the medium-run horizon, speaking directly our empirical findings. To do this, we generalize our framework such that the choice of δ imposes (17) for $\kappa > 1$ to ensure σ_d is unspanned in longer-maturity interest rates. Moreover, we enforce Assumption 3 (implying $\sigma_{\mathbb{P}} = \sigma^*_{\mathbb{P}}$, $\alpha_0 \approx -\alpha_1, d_0 \approx -d_1$) so the model can jointly deliver stylized facts at short and intermediate horizons.[12]

The following proposition generalizes the disconnect of ERRP to spot yields of some intermediate tenor $\overline{\kappa}$ and, as a result, short-term yields can have some relationship with exchange rates in line with the UIP puzzle:

Proposition 2 (Horizon Variation) *An increase in σ_d is associated with both: (i) an increase in medium horizon ERRP $\frac{d\mathbb{E}_t[rx^{FX}_{t+\kappa}]}{d\sigma_d} > 0$ and a steeper relative slope $\frac{dS_t}{d\sigma_d} > 0$, but is not*

[12] As shown by Lustig et al. (2019), carry-trade returns at long horizons are zero (and thus unpredictable) if there are no differences in the permanent innovations of SDFs across countries. Notably, Andrews, Colacito, Croce, and Gavazzoni (2024) find evidence that $rx^{CT\ (\infty)} = 0$ only because it is negative before and positive post-GFC. Consistently, Chernov and Creal (2020) argue the evidence for zero ERRP at long horizons appears weak.

reflected in $\overline{\kappa}-$maturity yields $\frac{dy_{t,\overline{\kappa}}}{d\sigma_d} = 0$, (ii) an increase in short horizon expected currency returns $\frac{d\mathbb{E}_t[rx^{FX}_{t+1}]}{d\sigma_d} > 0$ and a fall in the short rate differential $\frac{dy_{t,1}}{d\sigma_d} < 0$, if and only if:

$$\delta = \tfrac{1}{\kappa}d_0^2 + (d_0 + d_1)^2(\kappa - 1) \tag{24}$$

Additionally, (iii) in the long run, ERRP are zero.

Proof: See Appendix B.1. □

4.2 Endogenizing Hidden Factors via Domestically Incomplete Markets

Term-structure models like this are almost exclusively written under complete markets ,where there can only exist a unique SDF. Here, we show that our hidden (or unspanned) transitory factor can arise as an equilibrium outcome in a large class of incomplete-markets models. The generalized model is consistent with no-arbitrage pricing by the SDF m_{t+1} (18), but the structure of risk is constrained due to trade in assets by an additional investor with SDF $\tilde{m}_{t+1}$—ruled out if domestic financial markets were complete. This setup relates to models of incomplete markets and limited participation such as Guvenen (2009) and Marin and Singh (2024), as well as models of preferred habitat as in Gourinchas et al. (2022) and Greenwood et al. (2023), and carries implications for fully specified models.

Compared to the previous section, suppose there exists a second investor in the Home country with the SDF:[13]

$$\tilde{m}_{t+1} = \log\beta - \frac{1}{2}\sigma^2_{\mathbb{T}} + \alpha_0\epsilon_{\mathbb{T},t+1} + \alpha_1\epsilon_{\mathbb{T},t}$$

We then make the following adaption to Assumption 1:

Assumption 1′ *Both investors in the Home country ($m_{t,t+1}, \tilde{m}_{t,t+1}$) trade frictionlessly in domestic bonds with maturities $\kappa \in \tilde{\boldsymbol{\kappa}}$. Only the investor characterized by $m_{t,t+1}$ can additionally trade frictionlessly in Foreign assets.*

The combination of the additional investor and Assumption 1′ delivers the following proposition:

Proposition 3 (Incomplete Markets and Hidden Factors) *Suppose $\tilde{\boldsymbol{\kappa}} = 1$ such that only the risk-free rate is traded by both m and $\tilde{m}$. No arbitrage requires $\delta = d_0^2$. More generally, for any $\kappa \in \tilde{\boldsymbol{\kappa}}$ maturity traded, condition (24) is required to satisfy no-arbitrage.*

Proof: Consider the set of conditions $\mathbb{E}[M_{t,t+\kappa}] = \mathbb{E}[\tilde{M}_{t,t+\kappa}] = 1/R_t^{(\kappa)}\ \forall\ \kappa \in \tilde{\boldsymbol{\kappa}}$ which reflect risk-sharing between agents at horizon $\tilde{\kappa}$. Once again, under complete markets $\tilde{\boldsymbol{\kappa}} = \mathbb{R}$, which implies the mean (and variance) of all multi-horizon SDFs is equalized. □

[13]Notice, this is a normalization. More generally, we can consider $m^i_{t,t+1} = log(D^i_{t,t+1}M_{t,t+1})$ and interpret the expectations above as cross-sectional (see also Constantinides and Duffie, 1996b; Marin and Singh, 2024).

Intuitively, bonds traded by both investors only price risks which both investors face and agree upon. In the spirit of preferred-habitat models, $\epsilon_{d,t+1}$ can reflect investor-specific demand shocks, which drive both the term structure of interest rates and exchange rates. Since $\tilde{m}_{t+1}$ does not face these habitat shocks, the only equilibrium consistent with no arbitrage requires δ such that (17) holds.[14]

4.3 A Dynamic Asset-Pricing Model

Finally, we show our results generalize to a canonical dynamic asset-pricing model with stochastic volatility, building on a two-country version of the Cox et al. (1985) model, studied in Backus et al. (2001) and Lustig et al. (2019), amongst others. The purpose of this section is to show our results generalize to canonical asset pricing models in the literature, which have also been shown to capture key features of equilibrium models (e.g. Campbell and Cochrane, 1999; Bansal and Shaliastovich, 2013). We relegate derivations to the Appendix B.3.

Consider a representative Home investor's SDF which loads on three independent country-specific factors $z_{0,t}, z_{1,t}, z_{2,t}$:

$$\begin{aligned} -m_{t,t+1} &= z_{0,t} + (\lambda_1^2/2 - 1)z_{1,t} + \lambda_1\sqrt{z_{1,t}}\epsilon_{t+1} + (\lambda_2^2/2)z_{2,t} + \lambda_2\sqrt{z_{2,t}}\epsilon_{2,t+1}, \\ z_{i,t+1} &= (1-\phi_i)z_{2,t} + \phi_i z_{i,t} - \sigma_i\sqrt{z_{i,t}}\epsilon_{i,t+1}, \quad \text{for } i \in \{0,1\} \\ z_{2,t+1} &= (1-\phi_2)\theta \ + \phi_2 z_{2,t} - \sigma_2\sqrt{z_{2,t}}\epsilon_{2,t+1} \end{aligned} \tag{25}$$

where $\lambda_1, \lambda_2 < 0$ are coefficients which capture the price of risk with respect to each factor (see also, Backus et al., 1998, 2001; Lustig and Verdelhan, 2019). This setup resembles the model of central tendency in Balduzzi et al. (1998) where $z_{2,t}$ captures the long-run mean of the short rate.[15] To keep notation simple, we assume $z_{0,t}$ has a zero price of risk. The representative Foreign investor's SDF $m^*_{t,t+1}$ and country-specific pricing factors $z^*_{i,t}$ are defined analogously, and with symmetric coefficients ($\lambda^*_i = \lambda_i$, $\phi^*_i = \phi_i$, and $\sigma^*_i = \sigma_i$).

As is standard, assuming log-normality, combining (25) and the expression for the (log) price of an n-period bond, $p_{t,n} = \mathbb{E}_t[m_{t,t+1} + p_{t+1,n-1}] + (1/2)\text{var}_t(m_{t,t+1} + p_{t+1,n-1})$, we can write (log) bond prices as affine functions of pricing factors $p_{t,n} = -(\Omega_n + A_n z_{0,t} + B_n z_{1,t} + C_n z_{2,t})$, where Ω_n, A_n, B_n and C_n are recursively defined.

Hidden Factors in CIR. Once again, the SDF (25) can be interpreted as an equilibrium outcome from an incomplete markets model. Define a SDF $\tilde{m}_{t,t+1}$ which is identical to $m_{t,t+1}$

[14] Alternatively, domestic market incompleteness may arise because investors have heterogeneous expectations over the same fundamentals. To consider this, define a new subjective expectation operator $\tilde{\mathbb{E}}_t$. If investors agree on risks in the limit, markets are de-jure complete. In contrast, if there is disagreement, the second investor underestimates the d-factor. In this case, the only admissible equilibrium with trade in the one-period bond ($\tilde{\boldsymbol{\kappa}} = 1$) has a hidden factor, per Proposition 3. A further example would be a model of intermediation (e.g., Gabaix and Maggiori, 2015). Here, the SDF $m_{t,t+1}$ would additionally capture frictions (or different preferences) faced by the intermediary, not faced by the household $\tilde{m}_{t,t+1}$, i.e., the d- factor.

[15] Ang and Chen (2010) provide direct evidence for the importance of a time-varying long-run mean. Kozicki and Tinsley (2001) identify monetary policy and long run expectations as a good proxy for $z_{2,t}$, providing a supporting economic interpretation of this factor.

except that it does not depend on $z_{2,t}$—i.e., $\tilde{\lambda}_2 = 0$. If both investors trade the short bond, no-arbitrage requires $C_0 = 0$, which is satisfied in the model above. More generally, one can express the model with a conditional mean of $(\lambda_1^2/2 - 1)z_{1,t} + (\lambda_2^2/2 + \zeta)z_{2,t}$, leaving conditional variance unchanged. Then, ζ must be 0 to satisfy no arbitrage. Critically, however, there are additional requirements for subsequent C_n to be non-zero, such that the term structure captures information for $z_{2,t}$, specific to the dynamic model and detailed below.[16]

Term Structure. An immediate consequence of (25) is that short rates are given by $r_t^{(*)} = z_{0,t}^{(*)} - z_{1,t}^{(*)}$ and are countercyclical with respect to z_1, implying $B_n < 0$—consistent with Backus et al. (1998); Verdelhan (2010). Absent additional factors, this counterfactually implies a negative average slope of the yield curve and bond premium (Wachter, 2006) and increasingly negative longer-horizon UIP deviations (Lustig et al., 2019). The *ex ante* bond risk premium is given by:

$$\begin{aligned}\mathbb{E}_t\left[rx_{t,t+1}^{(\infty)}\right] + \frac{1}{2}\text{var}_t(r_{n,t+1}) &= -\operatorname{cov}_t(p_{t+1,n-1}, m_{t,t+1}) \\ &= \underbrace{-\lambda_1\sigma_1 B_{n-1}}_{<0} z_{1,t} - \lambda_2\sigma_2 C_{n-1} z_{2,t} \end{aligned} \tag{26}$$

and additionally depends on $\epsilon_{2,t}$ which is hidden from r_t.

The next proposition details necessary conditions for this framework to deliver our empirical findings.

Proposition 4 (Bond Yields and ERRP in CIR) *In the model described by* (25)*, a relatively steep Home yield-curve slope* ($S_t > S_t^*$) *can be associated with higher future ERRP* ($\mathbb{E}_t[rx_{t,t+\kappa}^{FX}]$)*, orthogonal to short-rate differentials, only if the loading on* $z_{2,t}$ *is positive at longer maturities* ($C_n > 0$) *which, in turn, requires* $z_{0,t}$ *to tend to* $z_{2,t}$*.*

Proof: See Appendix B.1. □

This proposition illustrates that our thesis—namely that factors hidden from spot yields but reflected in yield curves drive ERRP—is compatible with a larger class of dynamic models, but requires a model of central tendency. Within the dynamic framework, it demonstrates that condition (17) is compatible with other salient features of the data, such as a positive yield-curve slope and short-run UIP differentials. The sufficient condition to deliver $C_n > 0$, required for a positive yield-curve slope, is for $z_{2,t}$ to be persistent ($\phi_2 > \phi_1$)—which also rationalizes why the relative yield-curve slope especially matters at intermediate (as opposed to short or long) horizons.

Moreover, the model with time-varying volatility is sufficiently rich to speak to the regression evidence and account for a zero or near-zero R^2 coefficient of a regression of future exchange-rate changes (see Appendix B.3) on short-maturity interest rate differentials in the limit when $var(z_{2,t})$ is very high—while Proposition 4 implies a higher R^2 if the regression additionally includes yield-curve slopes which captures the hidden factor.

[16] Appendix B.3.1 generalizes hidden factors to longer tenors.

5 Conclusion

Overall, our paper highlights that a significant component of currency fluctuations and ERRP, at business-cycle horizons in particular, can be explained by cross-country differences in the term structure of interest rates. Preference-free results derived assuming no-arbitrage suggest this is driven by cross-country differences in the autocorrelation of investor valuations (SDFs) across countries, consistent with a notion of (transitory) business-cycle risk. To corroborate this, we turn to evidence that survey data on expectations for GDP and inflation explain relative yield-curve slopes. Then, regressing exchange-rate movements on the fitted component of the relative yield-curve slope, we find that cross-country asymmetries in macroeconomic expectations are a significant determinant of ERRP, orthogonal to interest rates, especially at 2 to 4-year horizons.

In addition to finding evidence that currency fluctuations reflect expectations of macroeconomic fundamentals, we illustrate the importance of transitory factors which can be 'hidden' from short-term interest rates because of offsetting effects on the conditional mean and variance of SDFs. As such, these factors are consistent with the literature on the exchange-rate 'disconnect', while still being captured by the yield-curve slope. Going a step further, we show that market incompleteness is a plausible mechanism which generates hidden factors as endogenous outcomes in equilibria with trade.

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Appendix

A Additional Information on Empirical Analysis

A.1 Data Sources

We use nominal zero-coupon government bond yields at maturities from 6 months to 10 years for 7 industrialised countries: US, Australia, Canada, Euro Area, Japan, Switzerland and UK. Our benchmark sample spans 1980:01-2024:09, although the panel of interest rates is unbalanced as bond yields are not available from the start of the sample in all jurisdictions. Table A1 summarizes the sources of nominal zero-coupon government bond yields, and the sample availability, for the benchmark economies in our study.

Table A1: Yield Curve Data Sources

Country	Sources	Start Date
US	Gürkaynak, Sack, and Wright (2007)	1971:11
Australia	Reserve Bank of Australia	1992:07
Canada	Bank of Canada	1986:01
Euro Area	Bundesbank (German Yields)	1980:01
Japan	Wright (2011) and Bank of England	1986:01
Switzerland	Swiss National Bank	1988:01
UK	Anderson and Sleath (2001)	1975:01

Notes: Data from before 1980:01 are not used in this paper.

Exchange-rate data is from *Eikon*, reflecting spot rates *vis-à-vis* the US dollar. Liquidity yields are from Du et al. (2018), available at the 1, 2, 3, 5, 7 and 10-year maturities. The earliest liquidity yields are available from 1991:04 for some countries (e.g., UK) and end in 2021:03. The latest liquidity yields are available from 1999:01 (e.g., euro area). For both exchange rates and liquidity yields, we use end-of-month observations.

A.2 Canonical and Augmented UIP Regressions

This appendix provides additional detail about our motivational analysis in Figure 1.

The canonical UIP regression for κ-month-ahead exchange-rate changes is:

$$e_{j,t+\kappa} - e_{j,t} = \beta_{1,\kappa}\left(r^*_{j,t,\kappa} - r_{t,\kappa}\right) + f_{j,\kappa} + u_{j,t+\kappa} \tag{A1}$$

where $e_{j,t} \equiv \log(\mathcal{E}_{j,t})$ is the (log) exchange rate of the Foreign country j *vis-à-vis* Home (base) currency at time t. It is defined as the Foreign price of a unit of base currency such that an increase in $e_{j,t}$ corresponds to a Foreign depreciation. $r^*_{j,t,\kappa}$ is the net κ-period continuously-compounded return in the Foreign country and $r_{t,\kappa}$ is the corresponding Home return. $f_{j,\kappa}$ is a country fixed effect and $u_{j,t+\kappa}$ is the disturbance.

Under the joint assumption of risk neutrality and rational expectations, the null hypothesis of UIP is that $\beta_{1,\kappa} = 1$ for all $\kappa > 0$ (and $f_{j,\kappa} = 0$ for all j and $\kappa > 0$).

To illustrate the link between exchange rates and the yield-curve slope in the simplest possible manner, we augment regression (A1) with the relative yield-curve slope $S^*_{j,t} - S_t$, estimating:

$$e_{j,t+\kappa} - e_{j,t} = \beta_{1,\kappa}\left(r^*_{j,t,\kappa} - r_{t,\kappa}\right) + \beta_{2,\kappa}\left(S^*_{j,t} - S_t\right) + f_{j,\kappa} + u_{j,t+\kappa} \tag{A2}$$

where $S^*_{j,t}$ is the slope of the Foreign-country-j yield curve at time t, and S_t is the slope of the base-country yield curve. $\beta_{2,\kappa}$ captures the relationship between the relative slope and exchange rates that is orthogonal to interest-rate differentials.

Along with its curvature, the level and slope of the yield curve are known to capture a high degree of variation in bond yields (Litterman and Scheinkman, 1991). We proxy the relative level in regression (A2) with the κ-period interest-rate differential $(r^*_{j,t,\kappa} - r_{t,\kappa})$. This ensures that the specification nests UIP, such that $\beta_{2,\kappa}$ captures the yield-curve slope's contribution *over and above* spot-yield differentials. Defining the *ex post* κ-period ERRP for Foreign currency as $rx^{FX}_{j,t,\kappa} \equiv r^*_{j,t,\kappa} - r_{t,\kappa} - (e_{j,t+\kappa} - e_{j,t})$ and combining with equation (A2) yields:

$$rx^{FX}_{j,t,\kappa} = (1 - \beta_{1,\kappa})\left(r^*_{j,t,\kappa} - r_{t,\kappa}\right) - \beta_{2,\kappa}\left(S^*_{j,t} - S_t\right) - f_{j,\kappa} - u_{j,t+\kappa} \tag{A3}$$

From this, we see that $\beta_{2,\kappa}$ can be interpreted as either the average Foreign depreciation (in percent) or the average *decrease* in the ERRP (in pp) associated with a 1pp increase in the slope of the Foreign yield curve relative to the US.

The estimated coefficients from Figure 1 are tabulated in columns (1)-(3) of Table A2. While the tent-shaped relationship is specific to using the US dollar as the base currency, the result is also robust to a number of changes to the specification, including: excluding spot-yield differentials from (A2), since these are themselves predicted by the yield curve slope (column (4)); and including the relative curvature as an extra regressor in (A2) (columns (5)-(7)).

A.3 Alternative Currency Bases

The tent-shaped pattern for the relative slope coefficient appears specific to the USD currency base. This is shown in Table A3, which plots the coefficients on the relative slope when regression (A2) is estimated with each alternative currency base in turn (i.e., AUD, CAD, CHF, EUR, JPY, GBP). For almost all currencies, the estimated coefficients on the relative are broadly insignificant at business-cycle horizons, and there is very little sign of a tent-shaped relationship with positive coefficients across horizons—except for the CHF.

B Theoretical Proofs and Derivations Appendix

B.1 Proofs to Propositions

Proof to Proposition 1. Holding Foreign factors volatilities constant ($\sigma_i^*,\ i \in \{\mathbb{T}, d, \mathbb{P}\}$), an increase in Home transitory volatility ($\sigma_{\mathbb{T}}$) is associated with a relatively steeper yield-curve slope (22), but is reflected in interest-rate differentials (21) for $\kappa = 1$. In contrast, permanent volatility ($\sigma_{\mathbb{P}}$) affects neither the yield-curve slope nor yield differentials. For the d-factor, the if condition follows directly by imposing $\delta = d_0^2$ such that condition (17) is enforced for the specific SDF (18), and then comparing (21), (22) and (23). The only if condition follows because when $\delta \neq d_0^2$, the d factor behaves like the transitory factor ($\mathbb{T}$) and there is no movement in factor volatility which drives yield-curve slopes and ERRP, but is not reflected in short-maturity interest-rate differentials. □

Proof to Proposition 2. $\delta = \frac{1}{\bar{\kappa}}\left(d_0^2 + (d_0 + d_1)^2(\kappa - 1)\right)$ imposes (17) for $\kappa = \bar{\kappa}$—i.e., at intermediate horizons. The equalization of the permanent risk factor ($\sigma_{\mathbb{P}} = \sigma_{\mathbb{P}}^*$), along with the purging of permanent risk from the transitory factors ($\alpha_0 \approx -\alpha_1, d_0 \approx -d_1$) ensures ERRP are zero in the long-run and so there is no predictability. The following derivatives can be calculated:

$$\frac{dS_t}{d\sigma_d^2} = \left(1 - \frac{1}{\kappa}\right)\frac{d_0^2}{2}, \quad \frac{dS_t}{d\sigma_{\mathbb{T}}^2} = \left(1 - \frac{1}{\kappa}\right)\frac{\alpha_0^2}{2} \tag{B1}$$

$$\frac{d\mathbb{E}_t[rx_{t+\kappa}^{FX}]}{d\sigma_d^2} = \frac{1}{\kappa}\frac{d_0^2}{2}, \quad \frac{d\mathbb{E}_t[rx_{t+\kappa}^{FX}]}{d\sigma_{\mathbb{T}}^2} = \frac{1}{\kappa}\frac{\alpha_0^2}{2} \tag{B2}$$

$$\frac{dy_{t,\kappa}}{d\sigma_d} = 0, \quad \frac{dy_{t,1}}{d\sigma_{\mathbb{T}}} = \left(1 - \frac{1}{\kappa}\frac{\alpha_0^2}{2}\right) \tag{B3}$$

□

Proof to Proposition 4. Assuming $\lambda_1, \lambda_2 < 0$, and given that $B_{n-1} < 0$, (26) delivers a positive bond premium only if $C_{n-1} > 0$ for $n > 1$. We require $C_0 = 0$ so that $z_{2,t}$ is unspanned

Table A2: Coefficient estimates from canonical UIP regression and regression augmented with relative yield-curve slope

Maturity	(1) Canonical UIP	(2) Augmented UIP	(3)	(4) Slope Only	(5) Augmented UIP + Curvature	(6)	(7)
κ	$r^*_\kappa - r_\kappa$	$r^*_\kappa - r_\kappa$	$S^* - S$	$S^* - S$	$r^*_\kappa - r_\kappa$	$S^* - S$	$C^* - C$
6m	-0.89	-0.33	0.43	0.62	-0.10	0.93	-0.80
	(0.58)	(0.87)	(0.58)	(0.38)	(0.84)	(0.66)	(0.71)
12m	-0.77*	-0.05	1.06	1.11**	0.17	1.74	-1.00
	(0.43)	(0.70)	(0.88)	(0.55)	(0.69)	(1.09)	(1.06)
18m	-0.62*	0.40	2.27**	1.73***	0.66	3.32***	-1.51
	(0.36)	(0.55)	(0.95)	(0.65)	(0.57)	(1.25)	(1.21)
24m	-0.42	0.60	3.04***	2.08***	0.99*	5.03***	-2.93*
	(0.33)	(0.50)	(1.07)	(0.74)	(0.52)	(1.43)	(1.49)
30m	-0.27	0.80*	3.98***	2.55***	1.25***	6.68***	-4.06**
	(0.29)	(0.45)	(1.13)	(0.76)	(0.46)	(1.51)	(1.71)
36m	-0.12	0.84**	4.37***	2.73***	1.26***	7.25***	-4.44***
	(0.26)	(0.39)	(1.12)	(0.77)	(0.41)	(1.53)	(1.68)
42m	0.07	0.88***	4.31***	2.51***	1.34***	7.97***	-5.80***
	(0.25)	(0.33)	(1.05)	(0.83)	(0.35)	(1.52)	(1.78)
48m	0.26	0.89***	3.89***	2.01**	1.32***	7.74***	-6.28***
	(0.25)	(0.29)	(1.08)	(0.96)	(0.29)	(1.58)	(1.95)
54m	0.48**	0.98***	3.54***	1.43	1.33***	7.05***	-5.88***
	(0.23)	(0.25)	(1.12)	(1.07)	(0.25)	(1.70)	(2.09)
60m	0.66***	1.08***	3.32***	1.01	1.34***	6.28***	-5.08**
	(0.22)	(0.25)	(1.20)	(1.17)	(0.23)	(1.82)	(2.18)
66m	0.84***	1.19***	3.15**	0.65	1.37***	5.36***	-3.90*
	(0.21)	(0.24)	(1.23)	(1.25)	(0.23)	(1.87)	(2.23)
72m	1.03***	1.30***	2.77**	0.13	1.42***	4.31**	-2.79
	(0.18)	(0.21)	(1.14)	(1.24)	(0.21)	(1.76)	(2.03)
78m	1.16***	1.33***	2.03**	-0.54	1.41***	3.26**	-2.27
	(0.16)	(0.19)	(1.01)	(1.17)	(0.19)	(1.60)	(1.93)
84m	1.21***	1.29***	1.11	-1.25	1.33***	1.97	-1.61
	(0.16)	(0.18)	(0.95)	(1.12)	(0.17)	(1.56)	(2.05)
90m	1.20***	1.22***	0.31	-1.82	1.23***	0.51	-0.39
	(0.15)	(0.17)	(0.92)	(1.12)	(0.16)	(1.55)	(2.16)
96m	1.13***	1.10***	-0.47	-2.31**	1.08***	-0.93	0.90
	(0.15)	(0.16)	(0.88)	(1.08)	(0.15)	(1.60)	(2.31)
102m	1.02***	0.96***	-1.14	-2.68**	0.93***	-2.08	1.86
	(0.16)	(0.16)	(0.94)	(1.11)	(0.15)	(1.62)	(2.35)
108m	0.92***	0.84***	-1.71*	-3.00***	0.81***	-3.06*	2.69
	(0.15)	(0.16)	(1.02)	(1.15)	(0.15)	(1.65)	(2.36)
114m	0.88***	0.80***	-1.89*	-3.09***	0.77***	-3.76**	3.76
	(0.16)	(0.17)	(1.04)	(1.15)	(0.15)	(1.54)	(2.30)
120m	0.87***	0.79***	-2.14**	-3.28***	0.76***	-4.19***	4.19*
	(0.16)	(0.17)	(1.04)	(1.17)	(0.16)	(1.40)	(2.17)

Notes: Column (1) presents results from canonical UIP regression (A1), a regression of κ-period exchange-rate change $\Delta^\kappa e_{t+\kappa}$ on the κ-period return differential $r^*_{t,\kappa} - r_{t,\kappa}$. Columns (2)-(3) present results from augmented regression (A2), using relative yield-curve slope $S^*_t - S_t$ as an additional regressor. Column (4) presents results when relative slope is only regressor. Columns (5)-(7) documents results when relative curvature is added as regressor in (A2). Regressions estimated using pooled end-of-month data for 6 currencies (AUD, CAD, CHF, EUR, JPY, GBP) against the USD from 1980:01 to 2024:09, including country fixed effects. The panel is unbalanced. $*$, $**$ and $***$ denote statistical significance at the 10%, 5% and 1% levels, respectively, using Driscoll and Kraay (1998) standard errors (reported in parentheses).

Table A3: Relative slope coefficient from augmented UIP regression using alternative base currencies

Base:	(1) AUD $S^* - S$	(2) CAD $S^* - S$	(3) CHF $S^* - S$	(4) EUR $S^* - S$	(5) GBP $S^* - S$	(6) JPY $S^* - S$
6 months	0.02	0.27	0.69*	-0.49	-0.64	0.77
	(0.52)	(0.38)	(0.40)	(0.77)	(0.64)	(0.48)
12 months	0.28	0.60	1.31**	-1.19	-0.90	1.33*
	(0.73)	(0.54)	(0.61)	(1.27)	(0.90)	(0.74)
18 months	0.40	0.60	1.86**	-1.27	-0.58	1.50*
	(0.77)	(0.65)	(0.74)	(1.43)	(0.95)	(0.91)
24 months	0.35	0.65	2.09***	-1.00	-0.47	1.64
	(0.77)	(0.70)	(0.76)	(1.27)	(1.01)	(1.01)
30 months	0.20	0.35	2.38***	-1.09	-0.36	1.71
	(0.82)	(0.76)	(0.74)	(1.14)	(1.05)	(1.10)
36 months	0.05	-0.23	2.32***	-1.86*	-0.63	1.25
	(0.89)	(0.90)	(0.72)	(1.12)	(1.14)	(1.20)
42 months	-0.28	-1.24	2.07***	-2.77***	-1.13	0.46
	(0.99)	(0.96)	(0.75)	(1.05)	(1.23)	(1.30)
48 months	-0.70	-2.03**	1.37*	-3.70***	-1.76	-0.81
	(1.01)	(0.98)	(0.82)	(1.01)	(1.24)	(1.40)
54 months	-0.52	-2.43***	0.58	-3.71***	-1.88*	-1.55
	(0.97)	(0.93)	(0.86)	(1.08)	(1.14)	(1.40)
60 months	0.01	-2.25**	-0.03	-2.76**	-1.78	-1.74
	(0.91)	(0.94)	(0.84)	(1.33)	(1.16)	(1.45)
66 months	0.59	-1.68*	-0.52	-1.14	-1.59	-1.21
	(0.90)	(0.98)	(0.80)	(1.70)	(1.15)	(1.50)
72 months	1.07	-1.44	-1.11	0.57	-1.31	-0.55
	(0.94)	(0.97)	(0.73)	(1.88)	(0.94)	(1.47)
78 months	1.30	-0.86	-1.57**	2.02	-1.25	0.54
	(1.06)	(0.94)	(0.72)	(1.77)	(0.86)	(1.36)
84 months	1.54	-0.34	-1.81**	2.91*	-0.97	1.97
	(1.09)	(0.97)	(0.75)	(1.64)	(0.81)	(1.21)
90 months	2.00*	-0.29	-2.00**	3.27**	-0.68	2.58**
	(1.09)	(0.95)	(0.82)	(1.53)	(0.77)	(1.13)
96 months	2.20*	-0.74	-2.37***	3.01**	-0.88	2.45**
	(1.14)	(0.92)	(0.85)	(1.48)	(0.81)	(1.10)
102 months	2.29*	-1.47	-2.70***	2.35	-1.16	2.35**
	(1.20)	(1.01)	(0.91)	(1.54)	(0.90)	(1.05)
108 months	2.24*	-2.16**	-3.25***	1.63	-1.65*	2.21**
	(1.21)	(1.10)	(0.98)	(1.72)	(0.89)	(1.10)
114 months	1.86	-2.48**	-3.58***	1.41	-2.11**	1.97*
	(1.19)	(1.19)	(1.01)	(1.87)	(0.84)	(1.17)
120 months	1.30	-2.90**	-3.91***	1.14	-2.74***	1.09
	(1.18)	(1.26)	(1.03)	(2.04)	(0.90)	(1.16)

Notes: Coefficients on relative yield curve slope from extended regression (A2), using relative yield-curve slope $S_t^* - S_t$ as an additional regressor, for different currency bases. Regressions estimated using pooled end-of-month data for 7 currencies (AUD, CAD, CHF, EUR, JPY, GBP, USD) from 1980:01 to 2019:12, including country fixed effects. The panel is unbalanced. $*$, $**$ and $***$ denote statistical significance at the 10%, 5% and 1% levels, respectively, using Driscoll and Kraay (1998) standard errors (reported in parentheses).

in short interest rates, inspecting the recursion for C_n (see Appendix B.3), $C_{n-1} > 0$ is only possible because $\mathbb{E}[z_{0,t+1}] = z_{2,t}$, such that the recursion for C_{n-1} depends on $A_{n-1} > 0$. $\square$

B.2 Derivations for Section 4.1

Consider equations (19)-(20), repeated below for convenience:

$$m^{\mathbb{T}}_{t,t+1} = \log\beta - \frac{1}{2}\gamma\sigma^2_{\mathbb{T}} + \alpha_0\epsilon_{\mathbb{T},t+1} + \alpha_1\epsilon_{\mathbb{T},t} \underbrace{-\frac{1}{2}\delta\sigma^2_d + d_0\epsilon_{d,t+1} + d_1\epsilon_{d,t}}_{\text{unspanned transitory factor}}$$

$$m^{\mathbb{P}}_{t,t+1} = -\frac{1}{2}\sigma^2_{\mathbb{P}} + \epsilon_{\mathbb{P},t+1}$$

Construct $m^{\mathbb{T}}_{t,t+\kappa} = \sum_{i=0}^{\kappa-1} m^{\mathbb{T}}_{t+i,t+i+1}$, $m^{\mathbb{P}}_{t,t+\kappa} = \sum_{i=0}^{\kappa-1} m^{\mathbb{P}}_{t+i,t+i+1}$ as follows:

$$\begin{aligned} m^{\mathbb{T}}_{t,t+\kappa} &= \kappa\log\beta - \frac{1}{2}\gamma\kappa\sigma^2_{\mathbb{T}} + \alpha_0\epsilon_{\mathbb{T},t+1} + \alpha_1\epsilon_{\mathbb{T},t} + \cdots + \alpha_0\epsilon_{\mathbb{T},t+\kappa} + \alpha_1\epsilon_{\mathbb{T},t+\kappa-1} \\ &\quad -\frac{1}{2}\delta\kappa\sigma^2_d + d_0\epsilon_{d,t+1} + d_1\epsilon_{d,t} + \cdots + d_0\epsilon_{d,t+\kappa} + d_1\epsilon_{d,t+\kappa-1}, \quad \text{(B4)} \\ m^{\mathbb{P}}_{t,t+\kappa} &= -\frac{1}{2}\kappa\sigma^2_{\mathbb{P}} + \epsilon^{\mathbb{P}}_{t+1}\cdots + \epsilon^{\mathbb{P}}_{t+\kappa} \quad \text{(B5)} \end{aligned}$$

These imply the following moments:

$$\begin{aligned} \mathbb{E}_t[m^{\mathbb{T}}_{t,t+\kappa}] &= \kappa\log\beta - \frac{1}{2}\gamma\kappa\sigma^2_{\mathbb{T}} + \alpha_1\epsilon_{\mathbb{T},t} - \frac{1}{2}\delta\kappa\sigma^2_d + d_1\epsilon_{d,t}, \\ \mathbb{E}_t[m^{\mathbb{P}}_{t,t+\kappa}] &= -\frac{1}{2}\kappa\sigma^2_{\mathbb{P}}, \\ \text{var}_t(m^{\mathbb{T}}_{t,t+\kappa}) &= \alpha_0^2\sigma^2_{\mathbb{T}} + (\alpha_0+\alpha_1)^2(\kappa-1)\sigma^2_{\mathbb{T}} + d_0^2\sigma^2_d + (d_0+d_1)^2(\kappa-1)\sigma^2_d, \\ \text{var}_t(m^{\mathbb{P}}_{t,t+\kappa}) &= \kappa\sigma^2_{\mathbb{P}} \end{aligned}$$

Combining (9) with the above yields:

$$\begin{aligned} -r_{t,\kappa} &= \kappa\log\beta - \frac{1}{2}\gamma\kappa\sigma^2_{\mathbb{T}} - \frac{1}{2}\kappa\sigma^2_{\mathbb{P}} + \frac{1}{2}\left\{\alpha_0^2\sigma^2_{\mathbb{T}} + (\alpha_0+\alpha_1)^2(\kappa-1)\sigma^2_{\mathbb{T}}\right\} \quad \text{(B6)} \\ &\quad + \frac{1}{2}\left\{d_0^2\sigma^2_d + (d_0+d_1)^2(\kappa-1)\sigma^2_d + \kappa\sigma^2_{\mathbb{P}}\right\}, \\ &= \kappa\log\beta + \frac{\sigma^2_{\mathbb{T}}}{2}\left(\alpha_0^2 - \gamma\kappa + (\alpha_0+\alpha_1)^2(\kappa-1)\right) + \frac{\sigma^2_d}{2}\left(d_0^2 - \gamma\kappa + (d_0+d_1)^2(\kappa-1)\right) \end{aligned}$$

Condition (21) then follows from $y_{t,\kappa} = \frac{1}{\kappa}r_{t,k}$.

B.2.1 Internationally Incomplete Markets

When markets are incomplete, we follow Backus et al. (2001) and Lustig and Verdelhan (2019) and consider a (log) exchange-rate process is given by $e_{t+1} - e_t = m_{t,t+1} - m^*_{t,t+1} + \eta_{t+1}$ where η_{t+1} is an incomplete-markets wedge.

Proposition D.1 (Term Structure and Market Incompleteness) *When international financial markets are incomplete ($\eta_{t+1} \neq 0$), relative yield-curve slopes are unaffected $cov_t(log(S_t^*) - \log(S_t), \eta_{t+1}) = 0$ and proj($\mathbb{E}_t[rx_{t+\kappa}^{FX}] \mid \tilde{\epsilon}$) is unchanged.*

Proof: Following Lustig and Verdelhan (2019), frictionless trade across borders in Home and Foreign bonds for $\kappa = 1$ (i.e., (8) and (12)) and Foreign analogs, implies:

$$-\mathbb{E}_t[\eta_{t+1}] = \frac{1}{2}\text{var}_t(\eta_{t+1}) + \text{cov}_t(m_{t+1}, \eta_{t+1}), \tag{B7}$$

$$\mathbb{E}_t[\eta_{t+1}] = \frac{1}{2}\text{var}_t(\eta_{t+1}) + \text{cov}_t(m_{t+1}^*, -\eta_{t+1}) \tag{B8}$$

Evaluating (8), (12) and Foreign analogs for $y_{t,\kappa}, y_{t,\kappa}^*$ for $\kappa > 1$, then $\text{cov}_t(y_{t,\kappa}, \eta_{t+1}) = 0\ \forall\kappa$ which delivers the first part of the proposition. Moreover, additionally using (16) implies $\text{cov}_t(\tilde{\epsilon}, \eta_{t+1}) = 0$, delivering the second part. □

The proposition above does not depend on the specific form of SDFs and shows that, while market incompleteness will affect the volatility and predictability of exchange rates, it will not alter the relationship we study.[17]

One example of market incompleteness is the presence of convenience or liquidity yields. As shown in Engel and Wu (2022) and Jiang et al. (2024), these increase the predictability of exchange rates. However, Corsetti, Lloyd, and Marin (2020) shows convenience yields increase the predictability of the incomplete-markets wedge η_{t+1}, but are uncorrelated with the relative slope as long as they are constant along the term structure.

B.3 Derivations for Section 4.3

We focus on a symmetric model with country-specific factors.

Bond-Pricing Recursions. First, consider the one-period bond, $n = 1$:

$$\begin{aligned} p_{t,1} &= \mathbb{E}_t\left[m_{t,t+1}\right] + \frac{1}{2}\text{var}_t\left(m_{t,t+1}\right) \\ &= z_{0,t} + \left(1 - \frac{\lambda_1}{2}\right) z_{1,t} - \frac{\lambda_2}{2} z_{2,t} + \frac{\lambda_1^2}{2} z_{1,t} + \frac{\lambda_2^2}{2} z_{2,t} \\ &= -z_{0,t} + z_{1,t} \end{aligned}$$

where the first line uses the expression for the bond price for $n = 1$, the conditional expectation of equation (25) is used in the second line, and the resulting expression is rearranged to yield the third line. The one-period risk-free yield $y_{t,1}$ is therefore given by:

$$y_{t,1} = z_{0,t} - z_{1,t}$$

[17] Lustig and Verdelhan (2019) discuss that market incompleteness is an unlikely candidate for resolving exchange-rate puzzles, although Marin and Singh (2024) show that international market incompleteness has stronger implications in the presence of within country idiosyncratic risk.

Next, consider the general n-period bond price:

$$
\begin{aligned}
p_{t,n} =& \mathbb{E}_t\left[m_{t,t+1} + p_{t+1,n-1}\right] + \frac{1}{2}\text{var}_t\left(m_{t,t+1} + p_{t+1,n-1}\right) \\
=& - z_{0,t} + \left(1 - \frac{1}{2}\lambda_1^2\right) z_{1,t} - \frac{1}{2}\lambda_2^2 z_{2,t} - \Omega_{n-1} - A_{n-1}z_{0,t+1} - B_{n-1}z_{1,t+1} - C_{n-1}z_{2,t+1} \\
& + \frac{1}{2}\text{var}_t\Big(\lambda_1\sqrt{z_{1,t}}u_{t+1} + \lambda_2\sqrt{z_{2,t}}u_{2,t+1} - A_{n-1}z_{0,t+1} - B_{n-1}z_{1,t+1} - C_{n-1}z_{2,t+1}\Big) \\
=& - z_{0,t} + \left(1 - \frac{1}{2}\lambda_1^2\right) z_{1,t} - \frac{1}{2}\lambda_2^2 z_{2,t} - A_{n-1}[(1-\phi_0)z_{2,t} + \phi_0 z_{0,t}] \\
& - B_{n-1}[(1-\phi_1)z_{2,t} + \phi_1 z_{1,t}] - C_{n-1}[(1-\phi_2)\theta + \phi_2 z_{2,t}] \\
& + \frac{1}{2}\Big(\lambda_1^2 z_{1,t} + \lambda_2^2 z_{2,t} + A_{n-1}^2\sigma_0^2 z_{0,t} + B_{n-1}^2\sigma_1^2 z_{1,t} + C_{n-1}^2\sigma_2^2 z_{2,t} - \lambda_1 B_{n-1}\sigma_1 z_{1,t} - \lambda_2 C_{n-1}\sigma_2 z_{2,t}\Big)
\end{aligned}
$$

where the second line uses the linear pricing equation and (25); and the third line uses the process for the factors. Rearranging,the recursions can be seen in the final line:

$$
\begin{aligned}
\Omega_n =& \Omega_{n-1} + C_{n-1}(1-\phi_2)\theta \\
A_n =& 1 + A_{n-1}\phi_0 - A_{n-1}^2\sigma_0^2 \\
B_n =& -1 + B_{n-1}\phi_1 + \frac{1}{2}\lambda_1\sigma_1 B_{n-1} - \frac{1}{2}\left(B_{n-1}\sigma_1\right)^2 \\
C_n =& A_{n-1}(1-\phi_0) + B_{n-1}(1-\phi_1) + \phi_2 C_{n-1} + \frac{1}{2}\lambda_2\sigma_2 C_{n-1} - \frac{1}{2}\left(C_{n-1}\sigma_2\right)^2
\end{aligned}
$$

with initial conditions $A_0 = C_0 = 0$, $B_0 = -1$.

Bond Excess Returns. The *ex ante* n-period bond excess return is defined as $\mathbb{E}_t[rx_{t,t+1}^{(n)}] = \mathbb{E}_t[p_{t+1,n-1} - p_{t,n} - y_{t,1}]$. This can be written as:

$$
\begin{aligned}
\mathbb{E}_t\left[rx_{t,t+1}^{(n)}\right] =& \mathbb{E}_t\left[p_{t+1,n-1} - p_{t,n} - y_{t,1}\right] \\
=& \mathbb{E}_t\left[-\Omega_{n-1} + \Omega_n - A_{n-1}z_{0,t+1} + A_n z_{0,t} - B_{n-1}z_{1,t+1} + B_n z_{1,t} - C_{n-1}z_{2,t+1} + C_n z_{2,t}\right] \\
& - z_{0,t} + z_{1,t} \\
=& C_{n-1}(1-\phi_2)\theta_2 - A_{n-1}\mathbb{E}_t[z_{0,t+1}] + A_n z_{0,t} - B_{n-1}\mathbb{E}_t[z_{1,t+1}] + B_n z_{1,t} \\
& - C_{n-1}\mathbb{E}_t[z_{2,t+1}] + C_n z_{2,t} - z_{0,t} + z_{1,t} \\
=& \left[-A_{n-1}\phi_0 + A_n - 1\right] z_{0,t} + \left[-B_{n-1}\phi_1 + B_n + 1\right] z_{1,t} + \\
& \left[-A_{n-1}(1-\phi_0) - B_{n-1}(1-\phi_1) - C_{n-1}\phi_2 + C_n\right] z_{2,t}
\end{aligned}
$$

where line 2 uses the pricing equation, line 3 uses the recursion for A_n defined above, and line 4 expands the conditional expectation of factors and collects like terms. Evaluating the expression above in the limit as $n \to \infty$ yields:

$$
\begin{aligned}
\mathbb{E}_t\left[rx_{t,t+1}^{(\infty)}\right] = & \quad \left[A_\infty(1-\phi_0) - 1\right] z_{0,t} + \left[B_\infty(1-\phi_1) + 1\right] z_{1,t} + \\
& \left[(1-\phi_2)C_\infty - A_\infty(1-\phi_0) - B_\infty(1-\phi_1)\right] z_{2,t}
\end{aligned}
\tag{B9}
$$

which can be rearranged as:

$$\mathbb{E}_t\left[rx^{(\infty)}_{t,t+1}\right] = \quad [A_\infty(1-\phi_0)]\,(z_{0,t}-z_{2,t}) + [B_\infty(1-\phi_1)]\,(z_{1,t}-z_{2,t}) + [C_\infty(1-\phi_2)]\,z_{2,t} - z_{0,t} + z_{1,t} \tag{B10}$$

The bond premium is driven by the distance of factors 0 and 1 from their long-run mean and from movements in the long run mean itself. From the recursion formulas:

$$\mathbb{E}_t\left[rx^{(\infty)}_{t,t+1}\right] = \left[-\frac{1}{2}\left(A_\infty\sigma_0\right)^2\right] z_{0,t} + \left[\frac{1}{2}\lambda_1\sigma_1 B_\infty - \frac{1}{2}\left(B_\infty\sigma_1\right)^2\right] z_{1,t} + \left[\frac{1}{2}\lambda_2\sigma_2 C_\infty - \frac{1}{2}\left(C_\infty\sigma_2\right)^2\right] z_{2,t}$$

Then, the *ex ante* bond risk premium is given by:

$$\begin{aligned}\mathbb{E}_t\left[rx^{(\infty)}_{t,t+1}\right] + \frac{1}{2}\mathrm{var}_t(r_{n,t+1}) &= -\,\mathrm{cov}_t(p_{t+1,n-1}, m_{t,t+1}) \\ &= -\,\lambda_1\sigma_1 B_{n-1} z_{1,t} - \lambda_2\sigma_2 C_{n-1} z_{2,t},\end{aligned}$$

recovering equation (26) in the main body. Factor zero does not appear because there is a zero price of risk.

Yield-Curve Slope and Bond Premium Approximation. The yield curve slope is defined as the difference between yields on n- and 1-period bonds:

$$S_{t,n} = y_{t,n} - y_{t,1} = \frac{1}{n}\left(\Omega_n + A_n + B_n z_{1,t} + C_n z_{2,t}\right) - z_{0,1} + z_{1,t}$$

Evaluating this expression in the limit as $n \to \infty$ yields:

$$S_{t,\infty} = C_\infty(1-\phi_2)\theta - z_{0,t} + z_{1,t}$$

which arises from the recursions for A_n, B_n and C_n, where A_n, B_n and C_n have a finite limit and Ω_n grows linearly. The approximation of the slope by the bond risk premium $S_{t,\infty} \approx \mathbb{E}_t[rx^{(\infty)}_{t,t+1}]$ is also verified within the CIR model. Over long enough samples, $\mathbb{E}_t[z_{0,t}] = \mathbb{E}_t[z_{1,t}] = \mathbb{E}_t[z_{2,t}] = \theta$, yielding the result.

Exchange Rates Under complete markets, (log) one-period exchange rate changes are determined as $\Delta e_{t+i,t+i+1} = m^*_{t+i,t+i+1} - m_{t+i,t+i+1}$. The expected one period exchange rate change is therefore given by:

$$\mathbb{E}_t[e_{t+1}] - e_t = \mathbb{E}_t[m_{t,t+1} - m^*_{t,t+1}] = -(z_{0,t} - z^*_{0,t}) + (1-\lambda_1^2/2)(z_{1,t} - z^*_{1,t}) + \lambda^2/2(z_{2,t} - z^*_{2,t})$$

We focus on conditional risk premia because our symmetric setup implies that unconditional risk premia $\mathbb{E}_t[rx^{FX}_{t+1}]$ are zero. The one-period ERRP can be derived by combining equations

(14) and (25), assuming complete markets:

$$\mathbb{E}_t[rx^{FX}_{t,t+1}] = (\lambda_1^2/2)\mathbb{E}_t[z_{1,t+1} - z^*_{1,t+1}] + (\lambda_2^2/2)\mathbb{E}_t[z_{2,t+1} - z^*_{2,t+1}] \tag{B11}$$

where an increase in $z_{1,t+1} - z^*_{1,t+1}$ leads to a fall in $r_{t+1} - r^*_{t+1}$ and an increase in conditional ERRP, as in Fama (1984). However, the ERRP also depends on the second factor $(z_{2,t} - z^*_{2,t})$ so long maturity bonds are a useful orthogonal predictor, as in Ang and Chen (2010). We investigate longer horizon currency returns below.

Transitory-Permanent Variation. We begin by showing how to eliminate permanent innovations in the model, such that ERRP in the long-run are zero—and therefore there is no long-riun predictability. If there are no permanent innovations, Alvarez and Jermann (2005) show this requires:

$$\mathbb{E}_t\left[rx^{(\infty)}_{t,t+1}\right] = \frac{1}{2}\text{var}_t(m_{t,t+1})$$

Since this must be true for any value of $z_{0,t}, z_{1,t}, z_{2,t}$, using (B9), this requires:

$$[A_\infty(1-\phi_0) - 1] = 0, \tag{B12}$$

$$[B_\infty(1-\phi_1) + 1] = \frac{1}{2}\lambda_1^2, \tag{B13}$$

$$[C_\infty(1-\phi_2) - B_\infty(1-\phi_1) - A_\infty(1-\phi_0)] = \frac{1}{2}\lambda_2^2 \tag{B14}$$

such that:

$$\mathbb{E}_t\left[rx^{(\infty)}_{t+1} - rx^{(\infty)\,*}_{t+1}\right] = (\lambda_1^2/2)[z_{1,t} - z^*_{1,t}] + (\lambda_2^2/2)[z_{2,t} - z^*_{2,t}] \tag{B15}$$

coinciding exactly with the one-period $\mathbb{E}_t[rx^{FX}_{t+1}]$.

Longer-Horizon Currency Movements. The κ-step-ahead exchange-rate change is then given by:

$$\mathbb{E}_t[e_{t,t+\kappa}] - e_t = \sum_{i=1}^{\kappa}\mathbb{E}_t[\Delta^1 e_{t+i}] = \tag{B16}$$

$$\left[\underbrace{\frac{1-\phi_0^\kappa}{1-\phi_0}(z^*_{0,t} - z_{0,t}) + (1-\phi_0)\sum_{i=0}^{\kappa-1}\sum_{j=0}^{i-1}\phi_0^j(z^*_{2,t+i-1-j} - z_{2,t+i-1-j})}_{\sum_{i=1}^{\kappa-1}\mathbb{E}_t[z^*_{0,t+i} - z_{0,t+i}]}\right]$$

$$+\frac{\lambda_1^2 - 1}{2}\left[\underbrace{\frac{1-\phi_1^\kappa}{1-\phi_1}(z^*_{1,t} - z_{1,t}) + (1-\phi_1)\sum_{i=0}^{\kappa-1}\sum_{j=0}^{i-1}\phi_1^j(z^*_{2,t+i-1-j} - z_{2,t+i-1-j})}_{\sum_{i=1}^{\kappa-1}\mathbb{E}_t[z^*_{1,t+i} - z_{1,t+i}]}\right]$$

$$+\frac{\lambda_2^2}{2}\underbrace{\frac{1-\phi_2^k}{1-\phi_2}(z_{2,t}^*-z_{2,t})}_{\sum_{i=1}^{\kappa-1}\mathbb{E}_t[z_{2,t+i}^*-z_{2,t+i}]}$$

Importantly, expected depreciations are stricly increasing in both factors $(z_{1,t}, z_{2,t})$ ceteris paribus. This implies higher relevance of the second factor at longer horizons through two chanels. First, because factor 2 is assumed more persistent. Second, because factor 1 tend to factor 2 in the long run.

B.3.1 General Formulation with Hidden Factors.

Finally, as in Section 4.1, we consider a generalization of the representative Home investor's SDF such that factor 2 can be hidden from longer maturity interest rates:

$$-m_{t,t+1}=z_{0,t}+(\xi+\lambda_1^2/2)z_{1,t}+\lambda_1\sqrt{z_{1,t}}\epsilon_{t+1}+(\zeta+\lambda_2^2/2)z_{2,t}+\lambda_2\sqrt{z_{2,t}}\epsilon_{2,t+1} \quad \text{(B17)}$$

The price of an $n-$period bond is given by

$$p_t^{(n)}=-z_{0,t}-(\xi+\lambda_1^2/2)z_{1,t}-(\zeta_{+}\lambda_2^2/2)z_{2,t}-\Omega_{n-1}-A_{n-1}z_{0,t}-B_{n-1}z_{1,t}-C_{n-1}z_{2,t}$$
$$+\frac{1}{2}\left\{\lambda_1^2 z_{1,t}+\lambda_2^2 z_{2,t}+A_{n-1}^2\sigma_0^2 z_{0,t}+B_{n-1}^2\sigma_1^2 z_{1,t}+C_{n-1}^2\sigma_2^2 z_{2,t}-\lambda_1\sigma_1 B_{n-1}z_{1,t}-\lambda_2\sigma_2 C_{n-1}z_{2,t}\right\} \quad \text{(B18)}$$

It follows that:

$$\begin{aligned}
\Omega_n &=\Omega_{n-1}+C_{n-1}(1-\phi_2)\theta\\
A_n &=1+A_{n-1}\phi_0-A_{n-1}^2\sigma_0^2\\
B_n &=\xi+B_{n-1}\phi_1+\frac{1}{2}\lambda_1\sigma_1 B_{n-1}-\frac{1}{2}\left(B_{n-1}\sigma_1\right)^2\\
C_n &=\zeta+A_{n-1}(1-\phi_0)+B_{n-1}(1-\phi_1)+\phi_2 C_{n-1}+\frac{1}{2}\lambda_2\sigma_2 C_{n-1}-\frac{1}{2}\left(C_{n-1}\sigma_2\right)^2
\end{aligned}$$

where $\Omega_0=0, A_0=1, B_0=\xi, C_0=\zeta$.

Consider, for example, the maturity $\kappa=2$:

$$\begin{aligned}
p_t^{(2)} \;=\; & -[\xi(1-\phi_2)\theta_2]-\left[(1+\phi_0)-\frac{1}{2}\sigma_0^2\right]z_{0,t}-\left[\xi(1+\phi_1)+\frac{1}{2}\lambda_1\xi\sigma_1-\frac{1}{2}\xi^2\sigma_1^2\right]z_{1,t} \quad \text{(B19)}\\
& -\left[(1-\phi_0)+\xi(1-\phi_1)-\zeta(1+\phi_2)-\frac{1}{2}\zeta^2\sigma_2^2+\lambda_2\zeta\sigma_2\right]z_{2,t}
\end{aligned}$$

therefore,

$$p_t^{(2)}=\Omega_2+A_2 z_{0,t}+B_2 z_{1,t}, \quad \text{if} \quad \zeta^2\sigma_2^2+\zeta(1+\phi_2-\lambda_2\sigma_2)+[\xi(1-\phi_1)+(1-\phi_0)]=0$$

which, once again imposes (17) and ensures factor $z_{2,t}$ is not reflected in the spot-yield differential.